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A CLASS NUMBER FORMULA OF IWASAWA'S MODULES

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§ 0. Introduction.

Let p be an odd prime number which will be fixed throughout the following. Let k be a finite extension of $\boldsymbol{Q}$ and k_∞ be the cyclotomic $\boldsymbol{Z}_p$-extension $k\boldsymbol{Q}_\infty$ of k, where $\boldsymbol{Q}_\infty$ is the unique $\boldsymbol{Z}_p$-extension of $\boldsymbol{Q}$ (c.f. [6]). For any $n \geqq 0$, let k_n be the unique extension of k in k_∞ of degree p^n over k: $k=k_0 \subset k_1 \subset \cdots \subset k_\infty$, and let $\Gamma_n = \mathrm{Gal}(k_\infty/k_n)$. Let A_n be the p-Sylow subgroup of the ideal class group of k_n and D_n be the subgroup of A_n consisting of ideal classes containing ideals $\prod \mathfrak{P}^{m(\mathfrak{P})}$, where $\mathfrak{P}$ runs over all primes of k_n lying over p and $m(\mathfrak{P}) \in \boldsymbol{Z}$. Let A'_n be the factor group A_n/D_n (c.f. [6]).

We assume that k is a CM field. Then k_∞ is also a CM field. Let j denote the complex conjugation of k_∞. For any $\boldsymbol{Z}[\{1, j\}]$-module M, let

$$M^- = \{a \in M \mid (1+j)a = 0\}.$$

(0.1) DEFINITION. Let $A_\infty^- = \varinjlim A_n^-$ and $A_\infty'^- = \varinjlim A_n'^-$, with respect to the natural maps induced from inclusion maps $k_n \to k_m$ for $m \geqq n \geqq 0$.

In [3] Greenberg, and in [2] Ferrero and Greenberg have proved that, if k is abelian over $\boldsymbol{Q}$, then the order of $(A_\infty'^-)^{\Gamma_n}$ is finite for any $n \geqq 0$. We shall compute its order by using p-adic L-functions associated to k when the degree of k over $\boldsymbol{Q}$ is prime to p.

In the following, we assume that k is a finite imaginary abelian extension of $\boldsymbol{Q}$ whose degree is prime to p. Let G denote the Galois group $\mathrm{Gal}(k/\boldsymbol{Q})$ and $\hat{G}$ be its character group $\mathrm{Hom}(G, \bar{\boldsymbol{Q}}_p^\times)$, where $\bar{\boldsymbol{Q}}_p$ is a fixed algebraic closure of $\boldsymbol{Q}_p$, We also consider $\hat{G}$ as the set of primitive Dirichlet characters with values in $\bar{\boldsymbol{Q}}_p$ which are associated to the extension $k/\boldsymbol{Q}$ by class field theory. Let ω be the Teichmüller character module p. Take $\phi \in \hat{G}$ with $\phi \neq \omega$ and $\phi(j) = -1$. Let $L_p(s; \omega\phi^{-1})$ be the p-adic L-function attached to $\omega\phi^{-1}$. For $\kappa \in 1+p\boldsymbol{Z}_p$ with $\kappa \notin 1+p^2\boldsymbol{Z}_p$, using Iwasawa's construction of p-adic L-functions, we have the unique power series $f(T; \omega\phi^{-1}) \in \Lambda_\phi$ such that

$$f(\kappa^{s-1}; \omega\phi^{-1}) = L_p(s; \omega\phi^{-1}),$$

where $\boldsymbol{Z}_p[\phi] = \boldsymbol{Z}_p[\{\text{all values of } \phi\}]$ and $\Lambda_\phi = \boldsymbol{Z}_p[\phi][[T]]$. We note that

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