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GROUP ACTIONS ON SPHERE BUNDLES OVER SPHERES

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Introduction.

In a previous paper [8], we studied group actions on S^k-bundles over S^n in cases of (1) $k<n\leqq 8$, (2) $k\geqq n$ and (3) $k=n-1$, $n\equiv 0 \bmod 4$. It is the purpose of this paper to estimate degrees of symmetry of bundle spaces and to construct group actions on semistable bundles over spheres.

Using Ku-Mann-Sicks-Su's theorems [5], [6], we shall give some estimation for upper bounds on degrees of symmetry in § 1. In § 2, we shall construct generators of stable groups $\pi_{4s-1}(SO)$ which yield group actions on bundle spaces. Some theorems due to Kervaire [4] provide an information on generators of semistable groups $\pi_i(SO(n))$. In § 3 we shall obtain some group actions on semistable sphere bundles over spheres. I would like to thank Professor S. Sasao for helpful conversations.

§ 1. Degree of symmetry.

For a closed connected smooth manifold M, the degree of symmetry of M denoted by $N(M)$ is defined as the upper bound of the dimensions of all compact Lie groups which act effectively and smoothly on M. Then we have next propositions.

Proposition 1. *Let B be an S^k-bundle over S^n, where $n\geqq 9$. Then B can not be a homotopy sphere.*

Proof. If B is a sphere, then by 28.2 and 28.6 in [9], we have $k=n-1$, and $n=1$, 2 or $n\equiv 0 \bmod 4$, where the key point is that a fibre S^k is contractible to a point in B, then we can replace the standard sphere in [9] by a homotopy sphere. Since $n=4s$ and $s\geqq 3$, the space B has a cell complex structure $B=S^{n-1}\cup_{2m\iota_{n-1}}\cup e^n \cup e^{2n-1}$ (cf. 3 in [7]), which is a contradiction.

Proposition 2. *Suppose that an S^k-bundle over S^n admits a cross section or $k<n-1$, then we have*

$$N(B)\leqq \frac{1}{2}n(n+1)+\frac{1}{2}k(k+1) \qquad \textit{for } n+k\geqq 19.$$

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