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KODAI MATH. J.
5 (1982), 482—494

SOME REMARKS ON THE RELATIVE GENUS FIELDS

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§ 1. Introduction.

Let k be a finite algebraic number field and K its finite extension. We denote by K^* the maximal abelian extension of k such that the composite field K^*K is unramified over K at all the finite or infinite primes, and the field K^*K is called the genus field of K with respect of k. (If K^* were defined as the maximal abelian extension of k such that K^*K was unramified over K at all the finite primes, the field K^*K was called the narrow genus field of K. We do not treat the narrow genus field in this paper.)

The field K^* is explicitly determined when k is the rational number field (see M. Ishida [5], [6] or M. Bhaskaran [1]). In § 3 of this paper we discuss the fundamental structure of K^* for general k. In § 4 we treat, as an example, the case of k=quadratic field of class number one in which 2 remains prime and $(K: k)=2$.

In § 5 we prove the following theorem; let k be a finite algebraic number field of class number one, G any finite abelian group, and m a positive integer such that $ex(G)|m$ and $m\|G|^{\infty}$. Then there exist infinitely many cyclic extensions F of k of degree m such that

$$C_F/C_F^{1-\sigma}\cong G(F^*/F)\cong G\,.$$

This paper contains the author's master thesis at Tokyo Institute of Technology (1981, March).

§ 2. Definitions.

Let k be a finite algebraic number field and K its finite extension. We denote by K^* the maximal abelian extension of k such that K^*K is unramified over K at all the finite or infinite primes. By the class field theory, K^* is the maximal abelian extension of k in the Hilbert class field of K, and $K^*\cap K$ is the maximal abelian extension of k in K. Throughout this paper the following notations are used;

O_k: the integer ring of k
U_k: the unit group of k

Received November 5, 1981