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A CERTAIN PROPERTY OF GEODESICS OF THE FAMILY OF RIEMANNIAN MANIFOLDS O_n^2 (V)

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Introduction.

This is exactly a continuation of Part (IV) ([14]) with the same title written by the present author. We shall use the same notation in it.

The period T of any non-constant solution $x(t)$ of the non-linear differential equation of order 2:

$$nx(1-x^2)\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+(1-x^2)(nx^2-1)=0 \tag{E}$$

with a constant $n>1$ such that $x^2+x'^2<1$ is given by the integral:

$$T=\sqrt{nc}\int_{x_0}^{x_1}\frac{dx}{x\sqrt{(n-x)\{x(n-x)^{n-1}-c\}}}, \tag{0.1}$$

where $0<x_0<1<x_1<n$ and $c=x_0(n-x_0)^{n-1}=x_1(n-x_1)^{n-1}$.

We proved in the last 10th section of Part (IV) that the following conjecture is true for $n\geqq 84$ and shall show that it is also true for $16\leqq n<84$ in the present paper.

CONJECTURE C. The period function T as a function of $\tau=(x_1-1)/(n-1)$ and n is monotone decreasing with respect to $n(>2)$ for any fixed τ $(0<\tau<1)$.

The section numbers of this paper will start from 11 for convenience sake and so the section numbers from 1 to 10 mean the ones in Part (IV).

§ 11. The fundamental principle to attain the purpose.

Setting $T=\Omega(\tau, n)$, we have the two formulas

$$\frac{\partial\Omega(\tau, n)}{\partial n}=-\frac{\sqrt{c}}{2b^2\sqrt{n}}\int_{x_0}^{1}\frac{(1-x)W(x, x_1)dx}{x(n-x)\sqrt{x(n-x)^{n-1}-c}} \tag{11.1}$$

where $b=\sqrt{B-c}$, $B=(n-1)^{n-1}$ and

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