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FIBRE HOMOTOPY SELF-EQUIVALENCES

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Introduction.

Let ξ be a fibre space $p: E\to E$ which means that the projection p has the COVERING HOMOTOPY PROPERTY for CW-complexes. Then a map $f: E\to E$ is a fibre preserving map if $p\circ f=p$ and a map f_0 is fibre homotopic to a map f_1 if there exists a homotopy $f_t: E\to E$ such that $p\circ f_t=p$ for all t. Now we call a fibre preserving map $f: E\to E$ a fibre homotopy self-equivalence if there is a fibre preserving map $g: E\to E$ such that $g\circ f$ and $f\circ g$ are both fibre homotopic to the identity 1_E. Then it is clear that the set of fibre homotopy classes of fibre homotopy self-equivalences forms a group under the multiplication defined by the composite of maps, which we denote by $\mathcal{L}(\xi)$. This group $\mathcal{L}(\xi)$ has been studied by several authors ([4], [5]) and also the purpose of this note is to investigate $\mathcal{L}(\xi)$ for ξ, a sphere bundle over a sphere. By using Gottlieb's theorem, K. Tsukiyama showed in a preprint that there exists a split extension:

$$0 \longrightarrow \pi_{n+q}(S^q) \longrightarrow \mathcal{L}(\xi) \longrightarrow Z_2 \longrightarrow 0$$

for ξ, a S^q-bundle over S^n $(n+2\leqq q)$. As a generalization of this result we prove

THEOREM A. *Let $\xi: S^q\to E\to S^n$ be a S^q-bundle over S^n $(n, q>2)$ with a cross-section, so that there exists $\eta\in\pi_{n-1}(SO(q))$ with $i_*(\eta)=\xi$. If $J(\eta)$ is contained in $\Sigma^2(\pi_{n+q-3}(S^{q-2}))$ we have an exact sequence·*

$$0 \longrightarrow \pi_{n+q}(S^q)/[\pi_{n+1}(S^q), \iota_q] \longrightarrow \mathcal{L}(\xi) \longrightarrow Z_2\# P_n^q \longrightarrow 0,$$

where P_n^q denotes the kernel of the homomorphism defined by Whitehead product $[\,, \iota_q]: \pi_n(S^q)\to\pi_{n+q-1}(S^q)$ and $\#$ denotes the semi-direct product with a relation $\tau\cdot b\cdot\tau=(-\iota_q)_* b$ for $\tau\neq 1\in Z_2$.

Moreover, as a bi-product of the proof of Theorem A, we obtain

THEOREM B. *If $J(\eta)\circ\Sigma^{q-1}\pi_{n+1}(S^n)\subset[\pi_{n+1}(S^q), \iota_q]$, then fibre homotopy self-equivalences $f_0, f_1: E\to E$ are fibre homotopic if and only if they are homotopic.*

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