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KILLING VECTOR FIELDS ON NON-COMPACT RIEMANNIAN MANIFOLDS WITH BOUNDARY*

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1. Introduction.

The study of Killing vector fields on compact Riemannian manifolds with boundary had been started by K. Yano [3]. In a previous paper [5], we discussed non-existence of Killing vector fields with finite global norms on complete Riemannian manifolds (without boundary).

The purpose of the present paper is to discuss non-existence of Killing vector fields with finite global norms on non-compact Riemannian manifolds with boundary.

We shall be in C^∞-category. Latin indices run from 1 to $n+1$ and Greek ones from 1 to n, and the Einstein summation convention will be used.

2. Riemannian manifold with boundary.

Let $\mathcal{M}$ be a complete, non-compact, connected and orientable Riemannian manifold of dimension $n+1$ and g (resp. ∇) the Riemannian metric (resp. the Riemannian connection) on $\mathcal{M}$. We take a non-compact manifold $\overline{M}=\partial M\cup M$ such that M is a noncompact, connected, open submanifold of $\mathcal{M}$ and $\partial M=\overline{M}-M$ is an n dimensional, compact, connected submanifold of $\mathcal{M}$, where $\overline{M}$ denotes the closure of M in $\mathcal{M}$. Then $\overline{M}$ is a Riemannian manifold with boundary ∂M, and the Riemannian metric on $\overline{M}$ is induced from the Riemannian metric g on $\mathcal{M}$. $\overline{M}$ is complete as a metric space with the distance determined by the induced Riemannian metric on $\overline{M}$. For simplicity, hereafter, we denote by g the induced Riemannian metric on $\overline{M}$ and by ∇ the Riemannian connection on $\overline{M}$.

At each point p of ∂M, there exists a coordinate neighborhood system $\{U; (x^i)\}$ of p in $\mathcal{M}$ such that $U\cap\overline{M}$ is represented by $x^{n+1}\geqq 0$ and $U\cap\partial M$ is represented by $x^{n+1}=0$. Such a coordinate neighborhood system is called a boundary coordinate system. And $\{U\cap\partial M; (x^\alpha)\}$ is the induced coordinate system on ∂M. If $\{U; (x^i)\}$ and $\{V; (y^i)\}$ are boundary coordinate systems satisfying $U\cap V\neq\emptyset$, then we have that

*) Dedicated to Professor Isamu Mogi on his 60th birthday.

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