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ON THE INVARIANT SUBMANIFOLD OF A *CR*-MANIFOLD

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§0. Introduction.

Differential geometry of Kaehler submanifolds has been studied from many points of view (see K. Ogiue [4], B. Smyth [7] etc.). On the other hand, *CR*-structures are recently developed by several authors (see Chern-Moser [1], Tanaka [8], [9], Webster [10], [11], S. Ishihara [2], Sakamoto-Takemura [5], [6] and so on). In [5], [6], the authors gave a change of canonical connections associated with almost contact structures belonging to a *CR*-structure and also derived the generalized Bochner curvature as a curvature invariant. The purpose of the present paper is to study invariant submanifolds of a *CR*-manifold and to prove some theorems similar to the Kaehler case.

In §1 we shall recall definitions and results given in [5], [6]. §2 well be devoted to the study of an invariant submanifold of a *CR*-manifold. In §3 we shall obtain some theorems similar to the Kaehler case. The author wishs to express his heartly thanks to Professors S. Ishihara and K. Sakamoto for their constant encouragement and valuable suggestions.

§1. Preliminaries.

In this section, we shall recall definitions and some properties of *CR*-structures for later use. Let $\mathcal{M}$ be a connected orientable C^∞-manifold of dimension $2n+1$ ($n \geqq 1$) and $(\mathcal{D}, J)$ a pair of a hyperdistribution $\mathcal{D}$ and a complex structure J on $\mathcal{D}$. The pair $(\mathcal{D}, J)$ is called a *CR-structure* if the following two conditions hold:

$$[JX, JY]-[X, Y]\in\Gamma(\mathcal{D}), \tag{1.1}$$

$$[JX, JY]-[X, Y]-J([X, JY]+[JX, Y])=0 \tag{1.2}$$

for every $X, Y\in\Gamma(\mathcal{D})$ where $\Gamma(\mathcal{D})$ denotes the set of all vector fields contained in $\mathcal{D}$. Let θ be a local 1-form annihilating the hyperdistribution $\mathcal{D}$. If the restriction of the 2-form $d\theta$ to $\mathcal{D}$ is nondegenerate, then the *CR*-structure $(\mathcal{D}, J)$ is called to be *nondegenerate*. In the sequel $(\mathcal{D}, J)$ will be always a nondegenerate *CR*-structure.

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