POISSON APPROXIMATION FOR SUMS OF
INDEPENDENT BIVARIATE
BERNOULLI VECTORS

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1. Introduction. It is well known fact as Poisson’s theorem that for a
given sequence of \(\{p_n, n \geq 1\}\) such that \(p_n \to 0\) \((n \to \infty)\) we have

\[
P_n(m) - \left(\frac{\lambda^n}{m!}\right)e^{-\lambda} \to 0 \quad \text{as} \quad n \to \infty
\]

for all non-negative integer \(m\) where

\[
\lambda_n = np_n, \quad P_n(m) = \binom{n}{m} p_n^m (1 - p_n)^{n-m}
\]

Furthermore, if \(np_n \to \lambda\) \((n \to \infty)\) then we have

\[
P_n(m) \to \left(\frac{\lambda^n}{m!}\right)e^{-\lambda} \quad \text{as} \quad n \to \infty.
\]

R. von Mises in the paper [3] has showed that if \(\{X_k, k \geq 1, j = 1, 2, \ldots, n_k\}\) is a sequence of independent random variables such that

\[
\max_{1 \leq j \leq n_k} p_{kj} \to 0, \quad \sum_{j=1}^{n_k} p_{kj} \to \lambda > 0 \quad (k \to \infty),
\]

then

\[
P\left[\sum_{j=1}^{n_k} X_{kj} = m\right] \to \left(\frac{\lambda^m}{m!}\right)e^{-\lambda} \quad (k \to \infty).
\]

In (1977) J. Mačys (see, [2]) has proved that the conditions (1.1) are necessary
as well.

Let \(\{(X_k, Y_k), k \geq 1\}\) be a sequence of random vectors bivariate Bernoulli
law, i.e.

\[
P[X_k = 0, Y_k = 0] = p_{00}, \quad P[X_k = 1, Y_k = 0] = p_{10},
\]

\[
P[X_k = 0, Y_k = 1] = p_{01}, \quad P[X_k = 1, Y_k = 1] = p_{11},
\]

where \(p_{00} + p_{10} + p_{01} + p_{11} = 1\).