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ON ENTIRE FUNCTIONS EXTREMAL FOR THE $\cos \pi\rho$ THEOREM HAVING PRESCRIBED ASYMPTOTIC GROWTH

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Introduction. If $f(z)$ is a nonconstant entire function, then Hadamard's three-circles theorem asserts that $\log M(r, f)$ is a convex, increasing function of $\log r$, where

$$M(r, f)=\max_{|z|=r} |f(z)| .$$

Hence, by well-known properties of logarithmically convex functions,

$$\log M(r, f)=\log M(r_0, f)+\int_{r_0}^{r} \frac{\Psi(t)}{t} dt \qquad (r \geqq r_0>0),$$

where $\Psi(t)$ is a nonnegative, nondecreasing function of t.

Valiron [6, p 130] showed the following result.

THEOREM A. *Let $\Lambda(r)$ be given by*

$$\Lambda(r)=\text{constant}+\int_{\alpha}^{r} \frac{\Psi(t)}{t} dt \qquad (r \geqq \alpha>0), \tag{1}$$

where $\Psi(t)$ is nonnegative, nondecreasing, and unbounded. Assume further that

$$\Lambda(r)<r^K, \tag{2}$$

for some $K>0$ and all sufficiently large r. Then there exists an entire function $f(z)$ such that

$$\log M(r, f) \sim \Lambda(r) \qquad (r \to \infty).$$

(In Theorem A, the hypothesis (2) can be ommitted. The proof is due to Clunie [1].)

If $f(z)$ is an entire function of order ρ (<1) and put

$$m^*(r, f)=\min_{|z|=r} |f(z)|,$$

then the classical $\cos \pi\rho$ theorem of Valiron and Wiman asserts that

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