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ON THE GROWTH OF SUBHARMONIC FUNCTIONS

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1. Introduction. Let $u(z)$ be a subharmonic function in the complex plane $\boldsymbol{C}$. We denote the order and lower order of $u(z)$ by ρ and μ, respectively. Let $M(r, u)$ and $m^*(r, u)$ denote the maximum and infimum of $u(z)$ on $|z|=r$, respectively. The classical $\cos \pi\rho$ theorem asserts that, given $\varepsilon>0$, the inequality

$$m^*(r, u)>(\cos \pi\rho-\varepsilon)M(r, u) \tag{1}$$

holds for a sequence $r=r_n\to\infty$, provided that $\rho<1$. Kjellberg [3] proved a striking improvement of this theorem.

THEOREM A. *If $\lambda\in(0, 1)$, then*

$$m^*(r, u)>\cos \pi\lambda\cdot M(r, u)$$

on an unbounded sequence of r, unless

$$r^{-1}M(r, u) \longrightarrow \alpha \qquad (r\to\infty),$$

where α is positive or ∞.

An important consequence of Theorem A is that if $\mu<1$ then the inequality (1) holds with ρ replaced by μ on an unbounded sequence of r. Another important consequence of Theorem A is the following fact.

"If $u(z)$ is subharmonic of order ρ $(0<\rho<1)$ and minimal type, then

$$m^*(r, u)>\cos \pi\rho\, M(r, u)$$

on a sequence of $r\to\infty$."

Such a fact does not always hold for subharmonic functions of order ρ and mean type. Barry [2] proved the following result.

THEOREM B. *Let $h(r)$ be positive and continuous for $r\geqq r_0$, and for each $s>0$,*

$$\frac{h(sr)}{h(r)} \longrightarrow 1 \qquad (r\to\infty).$$

Suppose that $h(r)\to 0$ $(r\to\infty)$ and $h'(r)>-O(r^{-1})$ $(r\to\infty)$. If $u(z)$ is subharmonic of order ρ $(0<\rho<1/2)$ and mean type, and

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