Y. WATANABE
KODAI MATH. J.
5 (1982), 329—338

KÄHLERIAN METRICS GIVEN BY CERTAIN SMOOTH POTENTIAL FUNCTIONS

BY YOSHIYUKI WATANABE

§1. Introduction.

As is well known, every Kählerian metric $ds^2=2\Sigma g_{\alpha\bar{\beta}}dz_\alpha d\bar{z}_\beta$ is locally expressible in the form

$$g_{\alpha\bar{\beta}}=\frac{\partial^2\phi}{\partial z_\alpha\partial\bar{z}_\beta},$$

with respect to local complex coordinates $\{z_\alpha\}$, $\alpha=1, \cdots, n$, where $\phi(z, \bar{z})$ is a real valued function of $\{z_\alpha, \bar{z}_\alpha\}$.

We now consider a Kählerian metric $g_{\alpha\bar{\beta}}$ with ϕ such that $\phi=f(t)$, $t=\Sigma z_\alpha\bar{z}_\alpha$, where $t\rightarrow f(t)\in C^\infty(R)$. S. S. Eum [1] studied such a Kählerian metric with non-zero constant holomorphic curvature defined in the complex number space C^n, and showed it is Fubinian, i. e.,

$$f(t)=\frac{1}{k}\log(kt+b)+c\,, \tag{1.1}$$

where k $(\neq 0)$, b (>0) and c are constant. A Kählerian manifold with constant holomorphic curvature is harmonic (cf. S. Tachibana [7]). For this reason, the present author [12] studied Kählerian manifolds, which are harmonic. On the other hand, P. F. Klembeck [2] has shown that the complex space C^n admits a complete Kählerian metric h with components

$$h_{\alpha\bar{\beta}}=\frac{\partial^2 f(t)}{\partial z_\alpha\partial\bar{z}_\beta},\quad f(t)=\int_0^\infty\frac{1}{r}\log(1+r)dr\,, \tag{1.2}$$

which has strictly positive curvature. The Kählerian manifold (C^n, h) is harmonic at the origin 0 of C^n (cf. §3). But the scalar curvature is not constant as we can directly compute. Therefore (C^n, h) is not harmonic, because a harmonic Riemannian manifold is Einsteinian (cf. [4]). Thus we are very interested in a Kählerian metric locally expressed in the form

$$g_{\alpha\bar{\beta}}=\frac{\partial^2 f(t)}{\partial z_\alpha\partial\bar{z}_\beta},\quad t\rightarrow f(t)\in C^\infty(R)\,, \tag{1.3}$$

Received May 25, 1981