S. KOBAYASHI
KODAI MATH. J.
5 (1982), 318—328

ON ANALYTIC CENTERS OF COMPACT SETS

BY SHŌJI KOBAYASHI

Introduction. In this paper we are concerned with a certain extremal problem involving second derivatives of bounded analytic functions at ∞, anlogous to the well-known extremal problem involving first derivatives that gives rise to the Ahlfors function, or to the Riemann map in the case of simply connected domains.

The concepts of analytic diameter and analytic center were first introduced by Vitushkin [10, 11] in the way of developping the theory of rational approximation. In 1974 F. Minsker [8] obtained their several properties mainly for planar continua and presented open problems on analytic diameters and analytic centers of general planar sets. Recently the author and N. Suita [7] investigated certain properties of analytic diameters, analytic centers and the associated extremal problems for planar compact sets, and also offered negative answers to the Minsker's problems except the fourth one which asks whether the set of analytic centers of a planar set is contained in its convex hull. In this paper we answer this Minsker's fourth problem affirmatively under a certain symmetric condition.

In Section 1 we list the definitions and notation which we use throughout this paper. In Section 2 we state preliminary known results as a series of lemmas for the convenience of citation. In Section 3 we prove our main theorems, one of which estimates the size of the analytic centers of a compact set when the compact set is contained in a closed disc, and the other when the compact set lies between two lines. In Section 4 we obtain the set of analytic centers as a exact form for simple symmetric compact sets on a line, which shows that the estimate given in [7, Theorem 2] by the author and Suita is sharp.

1. Definitions and notation. Let K be a compact set in the complex plane $\boldsymbol{C}$. We denote by $D=D(K)$ the unbounded component of the complement K^c of K. Let $AB(D)$ be the Banach space of all bounded analytic functions in D with the uniform norm $\|\cdot\|_\infty$, this is

$$\|f\|_\infty=\sup\{|f(z)| : z\in D\} \tag{1.1}$$

Received March 30, 1981