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THE VALUE-DISTRIBUTION OF RANDOM ENTIRE FUNCTIONS

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1. It is well-known that, for a given entire function $f(z)$, $\delta(a, f)=0$ $(a\in \boldsymbol{C})$ holds except possibly for a countable set, where "δ" denotes the deficiency and $\boldsymbol{C}$ the complex plane. We cannot generally remove the above exceptional set. The purpose of this paper is to show that the totality of entire functions $f(z)$ with $\delta^*(f)=\sup_{a\in C}\delta(a, f)>0$ is thin in a sense.

An open interval $\Omega=(-1/2, 1/2)$ is naturally a probability space. A Rademacher series $\varepsilon=(\varepsilon_k)_{k=1}^{\infty}$ in Ω is defined by $\varepsilon_k(\omega)=\mathrm{sign}(\sin 2^k\pi\omega)$ $(\omega\in\Omega)$. For a sequence $(a_k)_{k=1}^{\infty}$ $(\not\equiv 0)\subset \boldsymbol{C}$ with $\limsup_{k\to\infty}|a_k|^{1/k}=0$, a random entire function is defined by

$$(1)\qquad f_\varepsilon(z)=\sum_{k=1}^{\infty}\varepsilon_k a_k z^k=\left\{f_\omega(z)=\sum_{k=1}^{\infty}\varepsilon_k(\omega)a_k z^k\,;\ \omega\in\Omega\right\}.$$

A random entire function $f_\varepsilon(z)$ is a probability space of entire functions. We write simply $\delta(a, \omega)=\delta(a, f_\omega)$, $\delta^*(\omega)=\delta^*(f_\omega)$. In this paper, we shall show the following

THEOREM. *$\delta^*(\omega)=0$ almost surely (a.s.).*

2. We denote by "Pr" the probability. Put

$$(2)\qquad \begin{cases} T(r, f_\omega)=1/2\pi\displaystyle\int_0^{2\pi}\log^+|f_\omega(re^{it})|\,dt \\ T_0(r)=\log^+A_0(r)\,,\quad A_0(r)=\left(\displaystyle\sum_{k=1}^{\infty}|a_k|^2r^{2k}\right)^{1/2} \\ m(r, a, \omega)=1/2\pi\displaystyle\int_0^{2\pi}\log^+1/|f_\omega(re^{it})-a|\,dt \qquad (a\in \boldsymbol{C},\ r>0), \end{cases}$$

where $\log^+x=\max\{\log x, 0\}$ $(x>0)$. Note that $\delta(a, \omega)=\liminf_{r\to\infty} m(r, a, \omega)/T(r, f_\omega)$ $(a\in \boldsymbol{C}, \omega\in\Omega)$. If $\#\{k\,;\ a_k\neq 0\}<\infty$, then $f_\varepsilon(z)$ is a probability space of polynomials and we see easily $\delta^*(\omega)=0$ for all $\omega\in\Omega$. The proof in the case where

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