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ON THE HOMOTOPY OF TYPE *CW* COMPLEXES WITH THE FORM $S^2\cup e^4\cup e^6$

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§1. Introduction.

The purpose of this paper is to classify the homotopy type of *CW* complexes with the form $S^2\cup e^4\cup e^6$. For example, the total space of a sphere boundle over a sphere (or of a spherical fibration over a sphere) is a *CW* complex with the form $S^p\cup e^q\cup e^{p+q}$ up to homotopy. The homotopy type classification of such a complex was partially given by James and Whitehead [8] and Sasao [6], and for more general cases Toda considered. [7]

In general, it is not easy to find the complete invariants which determine the homotopy type of it. But we can find them in the case of *CW* complexes with the form $S^2\cup e^4\cup e^6$.

Let X be a *CW* complex with the form $S^2\cup e^4\cup e^6$, and $x_j\in H^{2j}(X, Z)$ be the generator for $j=1$, 2 or 3 such that,

$$(x_1)^2=m\cdot x_2 \quad\text{and}\quad x_1\cdot x_2=n\cdot x_3. \qquad (m, n\geqq 0)$$

Then we have

THEOREM 4.5. (a) *If m is odd, then*

$$Sq^2: H^4(X, Z_2)\longrightarrow H^6(X, Z_2)$$

is trivial and the homotopy type of X is uniquely determined by the pair of integers (m, n).

(b) *If m is even and*

$$Sq^2: H^4(X, Z_2)\longrightarrow H^6(X, Z_2)$$

is trivial, then the homotopy type of X is uniquely determined by the pair of integers (m, n).

(c) *If m is even and*

$$Sq^2: H^4(X, Z_2)\longrightarrow H^6(X, Z_2)$$

is non-trivial, then X has precisely two homotopy types which can be distingushed

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