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CANONICAL DECOMPOSITION OF HARMONIZABLE ISOTROPIC RANDOM CURRENTS

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Abstracts. First we show that the difference between harmonizable isotropic random currents and homogeneous isotropic ones is small in some sense. Next we obtain the quasi canonical decomposition of harmonizable isotropic random currents, and discuss the possibility of the canonical decomposition of them. Then we show that a certain kind of harmonizable isotropic random currents which is not homogeneous has the canonical decomposition.

§1. Introduction

Let $\mathfrak{L}_p$ be a class of random p-currents. We say that a random current U_p has the quasi canonical decomposition in $\mathfrak{L}_p$ if it has the unique decomposition $U_p=U_p^h+U_p^i+U_p^s$ in $\mathfrak{L}_p$ such that $dU_p^i=0$, $\delta U_p^s=0$ and $\Delta U_p^h=0$. The random currents U_p^i, U_p^s and U_p^h are called the irrotational, the solenoidal and the harmonic components of U_p respectively. If the covariances between any two components are zero in the quasi canonical decomposition, we call it the canonical decomposition. Physically the quasi canonical decomposition means the decomposition of a wave into the longitudinal one and the transversal one, and the canonical decomposition corresponds to the case where these two kinds of waves are stochastically independent.

Let $\mathfrak{U}_p$ be the class of homogeneous isotropic random p-currents, and $\mathfrak{W}_p$ be the class of harmonizable isotropic random p-currents. K. Ito (1956) has shown that every random current in $\mathfrak{U}_p$ has the canonical decomposition in it. In this paper we investigate the possibility of the canonical decomposition in $\mathfrak{W}_p$.

In Theorem 1 of §2 we have two characterizations of the class $\mathfrak{U}_p$ in the broader class $\mathfrak{W}_p$. The results may be understood as those stating that the difference between two classes is not large. Then we introduce a class $\mathfrak{V}_p$ of isotropic random currents which are superpositions of independent plane waves. This class stands between $\mathfrak{U}_p$ and $\mathfrak{W}_p$.

In §3 we first show that every random current in $\mathfrak{W}_p$ has the quasi canonical decomposition, but it is not necessarily the canonical one. Next we show

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