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CONTACT *CR* SUBMANIFOLDS

Dedicated to Professor Shigeru Ishihara
on his sixtieth birthday

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Introduction.

The *CR* submanifolds of a Kaehlerian manifold have been defined and studied by A. Bejancu [1] and are now being studied by many authors [3, 4, 5, 10, 11, 13, 14].

The main purpose of the present paper is to define what we call contact *CR* submanifolds of a Sasakian manifold and to study their properties [2, 13].

In §1, we first of all state some known results on submanifolds of a Sasakian manifold and define the contact *CR* submanifolds of a Sasakian manifold. We then prove a theorem which gives a necessary and sufficient condition in order for a submanifold tangent to the structure vector field ξ of a Sasakian manifold to be a contact *CR* submanifold.

§2 is devoted to the study of integrability conditions of the distributions defining contact *CR* structure of the contact *CR* submanifolds.

In §3, we deal with contact *CR* submanifolds of a Sasakian manifold whose normal connection is flat and in §4 we study minimal contact *CR* submanifolds of a Sasakian manifold.

§1. Submanifolds of Sasakian manifolds.

Let $\bar{M}$ be a $(2m+1)$-dimensional Sasakian manifold with structure tensors (ϕ, ξ, η, g). The structure tensors of $\bar{M}$ satisfy

$$\phi^2 X=-X+\eta(X)\xi\,, \qquad \phi\xi=0\,, \qquad \eta(\xi)=1\,, \qquad \eta(\phi X)=0\,,$$

$$g(\phi X, \phi Y)=g(X, Y)-\eta(X)\eta(Y)\,, \qquad \eta(X)=g(X, \xi)$$

for any vector fields X and Y on $\bar{M}$. We denote by $\bar{\nabla}$ the operator of covariant differentiation with respect to the metric g on $\bar{M}$. We then have

$$\bar{\nabla}_X\xi=\phi X\,, \qquad (\bar{\nabla}_X\phi)Y=\bar{R}(X, \xi)Y=-g(X, Y)\xi+\eta(Y)X\,,$$

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