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ON THE ALGEBRAIC STRUCTURES OF GRADED LIE ALGEBRAS OF SECOND ORDER

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§ 0. Introduction.

In 1973, by using of generalized Jordan triple systems of second order (=Kantor systems), I. L. Kantor [4] has given the models of graded Lie algebras of second order with involutive automorphism τ. In this note, we shall prove the converse, that is, if τ is an automorphism of a Lie triple system in a graded Lie algebra of second order such that $\tau^2=1$ (resp. -1), it characterizes the Kantor (resp. Freudenthal) system. We also give a simple connection between the two kinds of triple systems.

§ 1. A characterization of Kantor and Freudenthal systems.

We consider a graded Lie algebra of second order

$$\mathfrak{G}=\mathfrak{G}_{-2}\oplus\mathfrak{G}_{-1}\oplus\mathfrak{G}_0\oplus\mathfrak{G}_1\oplus\mathfrak{G}_2 \qquad \text{(direct sum)}$$
$$(1.1) \qquad [\mathfrak{G}_i, \mathfrak{G}_j]\subset\mathfrak{G}_{i+j}$$

over a field k of characteristic zero. Then the vector space $\mathfrak{G}_{-1}\oplus\mathfrak{G}_1$ becomes a Lie triple system (L. t. s.) with a triple product $[[X, Y], Z]$ where $[\ ,\ ]$ is the Lie product of $\mathfrak{G}$ and elements X, Y, Z are in $\mathfrak{G}_{-1}\oplus\mathfrak{G}_1$ (cf. [7]). Let τ be an automorphism of the L. t. s. $\mathfrak{G}_{-1}\oplus\mathfrak{G}_1$ with respect to the triple product. Then τ is called an *ε-structure* on $\mathfrak{G}_{-1}\oplus\mathfrak{G}_1$ ($\varepsilon=\pm1$) if $\tau^2=\varepsilon id$ and $\tau(\mathfrak{G}_{\pm1})=\mathfrak{G}_{\mp1}$.

Let V be a finite dimensional vector space over the field k. Then V is called a *Kantor* (resp. *Freudenthal*) *system* (cf. [4], [2], [8]) if V has a trilinear operation $\phi: V\times V\times V\to V$ such that

1) $[L(a, b), L(c, d)]=L(L(a, b)c, d)-\varepsilon L(c, L(b, a)d)$,
2) $K(K(a, b)c, d)=L(d, c)K(a, b)+\varepsilon K(a, b)L(c, d)$

for $a, b, c, d\in V$, where $L(a, b)c=\phi(a, b, c)$, $K(a, b)c=L(a, c)b-L(b, c)a$ and $\varepsilon=1$ (resp. -1).

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