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ON A MINIMAX FORMULA OF LEJA'S

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1. Introduction. Let D be a domain in the compact complex plane containing ∞. Leja [5] presented two discrete formulas for the Green's function of D (if it exists) with the logarithmic singularity at ∞ in terms of the points of ∂D. Pommerenke [9] later proved a hyperbolic version of one of these formulas—the one which involves the Fekete-Leja extremal points. We will prove here a hyperbolic version of the other formula, which is a minimax formula. We will also point out an extremal property of the points involved in this type of minimax formula. The author would like to thank Professor James A. Jenkins for introducing Leja's work to him and for many suggestions.

2. Notation and Known Results. The unit disc $\{|z|<1\}$ will be denoted by Δ. Given two complex numbers a and b such that $1-\bar{b}a\neq 0$, we let

$$[a, b]=(a-b)/(1-\bar{b}a). \tag{1}$$

Then $d(a, b)=|[a, b]|$ defines a metric in Δ. Let E in the following denote a given compact set in Δ. Then the capacity of E is defined as follows ([10]; see also [8, 4]). Let

$$V(x_0, x_1, \cdots, x_n)=\prod_{0\leq i<j\leq n}|[x_i, x_j]|, \qquad \text{for} \quad x=\{x_0, \cdots, x_n\}\subset E;$$

$$V_n=V_n(E)=\max_x V(x_0, \cdots, x_n)(x=\{x_0, \cdots, x_n\}\subset E) \quad \text{and}$$

$$v_n=v_n(E)=V_n^{2/[n(n+1)]}.$$

Then $\lim_{n\to\infty} v_n(E)$ exists and is called the *capacity* of E, denoted by ρ or $\rho(E)$.

Given $x=\{x_0, \cdots, x_n\}\subset E$ and $z\in\Delta$, we let

$$\delta_n^{(j)}(z; x)=\prod_{i\neq j}|[z, x_i]|, \qquad \text{for} \quad j=0, 1, \cdots, n, \quad \text{and} \tag{2}$$

$$\delta_n=\max_x\min_{0\leq j\leq n}\delta_n^{(j)}(x_j; x). \tag{3}$$

Then we have

THEOREM 1. *$\delta_n^{1/n}\to\rho$ as $n\to\infty$* ([8]; see also [4]).

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