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RIEMANNIAN METRICS ON PRINCIPAL CIRCLE BUNDLES OVER LOCALLY SYMMETRIC KÄHLERIAN MANIFOLDS

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0. Introduction.

In Riemannian Geometry, one of the most interesting problems is to find all Einsteinian manifolds. A. Besse has suggested the research for Einsteinian manifolds satisfying the condition (see [3], p. 165)

$$(*)\qquad R_{ipqr}R_j{}^{pqr}=\text{constant } g_{ij},$$

where $g=(g_{ij})$ is the Riemannian metric and $R=(R^i{}_{jkl})$ is the curvature tensor. Its typical examples are a locally symmetric spaces and a harmonic Riemannian manifold (cf. [13]). But the author [17] has recently shown that $Sp(2)/SU(2)$ of Berger (cf. [2]) is an Einsteinian manifold satisfying (*).

On the other hand, J. E. D'Atri and H. K. Nickerson has initiated a study of the Riemannian manifold whose local geodesic symmetries are volume-preserving (up to sign). In this paper, we call such a manifold a (locally) volume symmetric space. It has been studied by J. E. D'Atri and H. K. Nickerson ([5]. [6]), K. Sekigawa ([15]) and Y. Watanabe ([16]). This class of manifolds obviously includes harmonic Riemannian manifolds and locally symmetric spaces. Then we are interested in Einsteinian manifolds (especially Einsteinian manifolds satisfying (*)), which are volume symmetric.

In this paper, we consider the Riemannian metric $\tilde{g}(t)$ given on a principal circle bundle P over a Kählerian manifold M (cf. § 2). We show that if M is locally symmetric, then $(P, \tilde{g}(t))$ is locally homogeneous and volume symmetric. Especially we also remark that an Einsteinian metric is given on P in the case that M is Einsteinian.

In § 2, we define the Riemannian metric on a principal circle bundle over an n-dimensional Riemannian manifold and give the fundamental formulas. In § 3, we calculate the covariant derivative of the Riemannian curvature tensor by using the structure equations obtained in § 2. In § 4, we define a tensor field T of type (1, 2) on P and state its properties for later use. In § 5, M is assumed to be a locally symmetric Kählerian manifold. Then by using the equations obtained in sections 3 and 4, we show the main results. In the last section, an

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