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TWO RESULTS ASSOCIATED WITH SPREAD RELATION

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0. Introduction.

Let $u=u_1-u_2$ be nonconstant, where u_1 and u_2 are subharmonic in the plane. For such a function u, we will write

$$N(r, u)=\frac{1}{2\pi}\int_0^{2\pi} u(re^{i\theta})d\theta .$$

Then the Nevanlinna characteristic of u is defined by

$$T(r)\equiv T(r, u)=N(r, u^+)+N(r, u_2).$$

With this $T(r, u)$, the order and lower order of u are defined as follows:

$$\rho=\overline{\lim_{r\to\infty}}\frac{\log T(r, u)}{\log r} \qquad \text{(the order of } u\text{)},$$

$$\mu=\underline{\lim_{r\to\infty}}\frac{\log T(r, u)}{\log r} \qquad \text{(the lower order of } u\text{)}.$$

Further, we use the notation

$$\delta(\infty)\equiv\delta(\infty, u)=1-\overline{\lim_{r\to\infty}}\frac{N(r, u_2)}{T(r, u)} .$$

(Throughout this paper, $|E|$ denotes the one-dimensional Lebesgue measure of the set E.)

Assume that $\mu<\infty$. Then, Baernstein [3] proved the following inequality.

Spread Relation: For any fixed $b\in(-\infty, \infty)$,

$$\overline{\lim_{r\to\infty}}|\{\theta : u(re^{i\theta})>b\}|\geqq\min\left\{\frac{4}{\mu}\sin^{-1}\left(\frac{\delta(\infty)}{2}\right)^{1/2}, 2\pi\right\} .$$

In connection with this relation, we show the following result.

THEOREM 1. *Let $u=u_1-u_2$ be nonconstant, where u_1 and u_2 are subharmonic in the plane. Suppose $\mu<\infty$ and $\delta(\infty)>0$. Let λ be a number satisfying*

$$\lambda>\mu , \quad \frac{4}{\lambda}\sin^{-1}\left(\frac{\delta(\infty)}{2}\right)^{1/2}\leqq 2\pi .$$

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