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ON ALMOST CONTACT METRIC COMPOUND STRUCTURE

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Introduction. K. Yano and U.-H. Ki [8] have recently introduced the notion of $(f, g, u, v, w, \lambda, \mu, \nu)$-structure in an odd-dimensional manifold M, which is an abstraction of the induced structure in a submanifold of codimension 3 in an almost Hermitian manifold, and studied conditions for such a structure to define an almost contact structure in M and properties of pseudo-umbilical submanifold of codimension 3 satisfying the conditions in a Euclidean space of even-dimension.

In the present paper, we shall introduce in § 1 the notion of metric compound structure in a manifold M of dimension m, which is a generalization of $(f, g, u, v, w, \lambda, \mu, \nu)$ and naturally induced in M if M is a submanifold in an almost Hermitian manifold $\tilde{M}$ of dimension n. In § 2, we shall seek for conditions in order that a metric compound structure defines an almost contact metric structure in M. After the definition of normality in § 3, we shall consider in § 4 submanifolds having a normal contact metric compound structure in a Kaehlerian manifold. In § 5, we shall disscuss properties and give geometrical characterization of pseudo-umbilical submanifolds in a Euclidean space. In § 6, we shall show that a metric compound structure possessing another property gives an almost contact metric structure.

Throughout this paper, we put $l=n-m$ and indices run the following ranges respectively:

$$\kappa, \lambda, \mu, \nu, \cdots = 1, 2, \cdots\cdots\cdots\cdots, n;$$

$$h, i, j, k, \cdots = 1, 2, \cdots, m;$$

$$p, q, r, s, \cdots = m+1, m+2, \cdots, n;$$

$$A, B, C, D, \cdots = 1, 2, \cdots, m, m+1, \cdots\cdots, n.$$

§ 1. Metric compound structure

Let $\tilde{M}$ be an n-dimensional almost Hermitian manifold and $(G, \tilde{F})$ the almost Hermitian structure, where G is the almost Hermitian metric and $\tilde{F}$ the almost complex structure of $\tilde{M}$. We denote by $G_{\lambda\mu}$ and $\tilde{F}_{\lambda}{}^{\kappa}$ components of G and $\tilde{F}$ with respect to a local coordinate system (x^{κ}). If $I=(\delta_{\lambda}{}^{\kappa})$ indicates the identity

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