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SOME ISOSPECTRAL PROBLEMS

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§ 0. Introduction

In the previous paper [7], we proved the following theorem.

THEOREM 0.1. *Let $M^{2n+1}(c)$ be a compact Sasakian space from of a constant ϕ-sectional curvature c of dimension $2n+1=5, 7, 9, 11$ or 13, and let M^* be a compact Sasakian manifold. If $M^{2n+1}(c)$ and M^* are isospectral with respect to the Laplace operator, then M^* is a $2n+1$ dimensional Sasakian space form of a constant ϕ-sectional curvature $c^*=c$. ($c\neq 31$ when $2n+1=13$)*

In this paper, we study isospectral problems not only of Sasakian manifolds but of their submanifolds. In addition, by using the similar methods, we study isospectral problems of quaternion Kaehler manifolds and some submanifolds of Kaehler manifolds.

§ 1. Special Classes of Sasakian Manifolds

Let $M^{2n+1}(c)$ be a $2n+1$ ($\geqq 5$) dimensional Sasakian manifold with structure tensors (ϕ, ξ, η). In [7] we proved the following two propositions, which give the characterizations of *Sasakian space forms* and *C-Einstein manifolds* in terms of equalities involving curvature tensors.

PROPOSITION 1.1. *A Sasakian manifold M^{2n+1} satisfies an inequality*

$$|R|^2 \geqq \frac{2}{n(n+1)} S^2 - \frac{4(3n+1)}{n+1} S + \frac{4n(3n+1)(2n+1)}{n+1}, \tag{1.1}$$

where S is the scalar curvature of M^{2n+1}. Equality holds if and only if M^{2n+1} is a Sasakian space form.

PROPOSITION 1.2. *A Sasakian manifold M^{2n+1} satisfies an inequality*

$$|\mathrm{Ricci}|^2 \geqq \frac{(S-2n)^2}{2n} + 4n^2. \tag{1.2}$$

Equality holds if and only if M^{2n+1} is a C-Einstein manifold.

See [7] for notations and definitions of Sasakian manifolds.
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