I. AMEMIYA AND T. ITO
KODAI MATH. J.
4 (1981), 418—425

WEAKLY NULL SEQUENCES IN JAMES SPACES ON TREES

Dedecated to Professor Goro Azumaya on his sixtieth birthday

BY ICHIRO AMEMIYA AND TAKASHI ITO

Introduction. R. C. James [2] and J. Lindenstrauss and C. Stegall [3] gave the examples of separable Banach spaces having no subspace isomorphic to l^1 whose duals are non-separable. We are concerned here with James' example. In [2], he constructed a Banach space having properties a) it is separable and its dual is non-separable and b) every infinite dimensional subspace contains a subspace isomorphic to l^2. Property a) is a direct consequence of his construction, but to see property b) requires a rather deep observation. Property b) is equivalent to

b′) for any weakly null normalized sequence $\{x_n\,;\,n=1, 2, \cdots\}$ there is a sequence $\{y_n\,;\,n=1, 2, \cdots\}$ equivalent to an l^2-basis for which each y_n is a linear combination of x_n's together with

b″) every infinite dimensional subspace contains a weakly null normalized sequence.

In this paper we will prove a stronger property than b′), namely that there is a *subsequence*, instead of linear combinations, of $\{x_n\,;\,n=1, 2, \cdots\}$ which is equivalent to an l^2-basis. In fact, we will show this under an (apparently) weaker assumption than being weakly null. It should be mentioned here that if we use H. P. Rosenthal's characterization of Banach spaces containing l^1 [5], property b″) is equivalent to saying that there is no subspace isomorphic to l^1.

In section 1, we give a definition of James spaces on trees, which are slightly more general than James' example, and we formulate our main result in Theorem. In section 2 we prove our main result.

§ 1. James Spaces and the Main Result.

Let T be a union of a countable family of pairwise disjoint non-empty finite sets P_n, $n=0, 1, 2, \cdots$. We call a point t of P_n a point of *level* n, and write $l(t)=n$. We assume there is a binary relation between points of P_n and points of P_{n+1}, which we call a *connection*, such that for every $n=0, 1, 2, \cdots$, each point of level n is connected to at least one point of level $n+1$ and each point of level $n+1$ is connected to only one point of level n. The following illustrates an example of connections between points of the first three levels

Received March 26, 1980