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THE SPECTRUM OF THE LAPLACE OPERATOR FOR A SPECIAL RIEMANNIAN MANIFOLD

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1. Introduction. Let (M, g) be a compact orientable Riemannian manifold of dimension n. Let $\Lambda^q(M)$ be the vector space of exterior q-forms on M, where $q=0, 1, \cdots, n$. We denote by $S_p^q(M, g)$ the spectrum of Δ on $\Lambda^q(M)$.

It was the following open problem. Does $S_p^q(M, g)$ determine the geometry of the Riemannian manifold (M, g)? The answer to this problem in general case is negative. This is a consequence of the counter example which is given in ([3]). If the Riemannian manifold (M, g) is a special one, then problem remains open.

It has been proved ([4]) that the three spectrums $S_p^0(S^n, g_0)$, $S_p^1(S^n, g_0)$ and $S_p^2(S^n, g_0)$ determine completely the geometry of the standard sphere (S^n, g_0).

One of the results of the present paper is to prove that for each standard sphere (S^n, g_0) there is at least one integer $q\in[0, n]$ such that the spectrum $S_p^q(S^n, g_0)$ determines completely the geometry on the sphere (S^n, g_0).

In the second paragraph we give some known results for the spectrum of the Laplace operator Δ which acts on the vector space $\Lambda^q(M)$, where $q=0, 1, \cdots, n$.

The spectrum of the Laplace operator on the $\Lambda^q(M)$, when the Riemannian manifold (M, g) has constant sectional curvature different from zero, is studied in the third paragraph.

2. We consider a compact, orientable Riemannian manifold (M, g) of dimension n. Let $\Lambda^q(M)$ be the vector space of all exterior q-forms on M, where $q=0, 1, \cdots, n$. For $q=0$, we obtain the set $\Lambda^0(M)$ of all differentiable functions on M.

Let $\Delta=-(d\delta+\delta d)$ be the Laplace operator which acts on the exterior algebra of M

$$\Lambda(M)=\Lambda^0(M)\oplus\Lambda^1(M)\oplus\cdots\oplus\Lambda^n(M)=\bigoplus_{q=0}^{n} \Lambda^q(M)$$

as follows

$$\Delta: \Lambda(M) \longrightarrow \Lambda(M), \quad \Delta: \Lambda^q(M) \longrightarrow \Lambda^q(M),$$

$$\Delta: \alpha \longrightarrow \Delta(\alpha)=-(d\delta+\delta d)(\alpha)=-d\delta(\alpha)-\delta d(\alpha), \ \forall\alpha\in \Lambda^q(M).$$

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