B.E. RHOADES
KODAI MATH. J.
4 (1981), 293—297

A FIXED POINT THEOREM FOR ASYMPTOTICALLY NONEXPANSIVE MAPPINGS

By B. E. Rhoades

Let H be a real Hilbert space, C a closed convex subset of H, T a selfmapping of C. Let $A_n x$ denote the nth term of the Cesàro transform of the sequence of iterates $\{T^k x\}$. Baillon [1] proved that, if T is a nonexpansive selfmapping of C which has a fixed point, then $\{A_n x\}$ converges weakly to a fixed point of T. This result has been extended to strongly regular matrices by Brézis and Browder [2], Bruck [3], and Reich [9]. In a recent paper [6] the Baillon result was extended to asymptotically nonexpansive mappings. In this paper the result of [6] is extended to a wide class of matrix methods.

A mapping T is said to be asymptotically nonexpansive over C if, for each $x, y \in C$,

$$\|T^i x - T^i y\| \leqq (1+\alpha_i)\|x-y\|, \qquad i=1, 2, \cdots, \tag{1}$$

where $\lim_i \alpha_i = 0$.

An infinite matrix $A=(a_{nk})$ is called regular if it is limit-preserving over c, the space of convergent sequences. Necessary and sufficient conditions for regularity are: (i) $\|A\|=\sup_n \sum_{k=0}^{\infty} |a_{nk}| < \infty$; (ii) $\lim_n a_{nk}=0$ for $k=0, 1, 2, \cdots$, and (iii) $\lim_n t_n = 1$, where $t_n=\sum_{k=0}^{\infty} a_{nk}$. Let X be a locally convex space. A sequence $\{x_n\} \subset X$ is said to be almost convergent, written ac, if there exists a point $s \in X$ such that $\lim_n \sum_{k=0}^{n-1} x_{k+i}/n = s$, uniformly in i. A matrix A will be called strongly regular if, in addition to satisfying conditions (i) and (iii) for regularity, it also satisfies (ii′) $\lim_n \sum_k |a_{nk} - a_{n,k+1}| = 0$. A is called triangular if all its entries above the main diagonal are zero.

THEOREM. *Let C be a closed convex subset of a real Hilbert space H, T an asymptotically nonexpansive selfmap of C such that $\{T^n z\}$ is bounded for each $z \in C$. Let A be a strongly regular matrix. Define $A_n x = \sum_{k=0}^{\infty} a_{nk} T^k x$. Then, for each $x \in C$, $\{A_n x\}$ converges weakly to a fixed point p, which is the asymptotic center of $\{T^n x\}$.*

The proofs of Lemmas 2 and 3 of [6] are independent of the matrix A involved. So, to prove the Theorem, it is sufficient to show that Lemma 1 of [6] is true for each strongly regular matrix A; i.e., there exists a positive integer

Received February 28, 1980