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NONLINEAR CONSTRACTIONS IN ABSTRACT SPACES

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I. Introduction.

Recently, Eisenfeld J. & Lakshmikantham V. [4, 5, 6], Bolen J. C. & Williams B. B. [1], Heikkila S. & Seikkala S. [7, 8], Chung K. J. [3], Kwapisz M. [10] and Ważewski T. [11] proved some fixed point theorems in abstract cones which extend and generalize many known results. In this paper, we extend some main results of Boyd D. W. & Wong J. S. W. [2] to cone-valued metric spaces.

II. Definitions.

Let E be a normed space. A set $K\subset E$ is said to be a cone if (i) K is closed (ii) if $u, v\in K$ then $\alpha u+\beta v\in K$ for all $\alpha, \beta\geqq 0$, (iii) $K\cap(-K)=\{\mathcal{O}\}$ where $\mathcal{O}$ is the zero of the space E, and (iv) $K^0\neq\emptyset$ where K^0 is the interior of K. We say $u\geqq v$ if and only if $u-v\in K$, and $u>v$ if and only if $u-v\in K$ and $u\neq v$. The cone K is said to be strongly normal if there is $\delta>0$ such that if $z=\sum_{i=1}^{n} b_i x_i$, $x_i\in K$, $\|x_i\|=1$, $\sum_{i=1}^{n} b_i=1$, $b_i\geqq 0$ implies $\|z\|>\delta$. The cone K is said to be normal if there is $\delta>0$ such that $\|f_1+f_2\|>\delta$ for $f_1, f_2\in K$ and $\|f_1\|=\|f_2\|=1$. The norm in E is said to be semimonotone if there is a numerical constant M such that $\mathcal{O}\leqq x\leqq y$ implies $\|x\|\leqq M\|y\|$ (where the constant M does not depend on x and y).

Let X be a set and K a cone. A function $d: X\times X\to K$ is said to be a K-metric on X if and only if (i) $d(x, y)=d(y, x)$, (ii) $d(x, y)=\mathcal{O}$ if and only if $x=y$, and (iii) $d(x, y)\leqq d(x, z)+d(z, y)$. A sequence $\{x_n\}$ in a K-metric space X is said to converge to x_0 in X if and only if for each $u\in K^0$ there exists a positive integer N such that $d(x_n, x_0)\leqq u$ for $u\geqq N$. A sequence $\{x_n\}$ in X is Cauchy if and only if for each $u\in K^0$ there exists a positive integer N such that $d(x_n, x_m)\leqq u$ for $n, m\geqq N$. The K-metric space (X, d) is said to be complete if and only if every Cauchy sequence in X converges.

Throughout the rest of this paper we assume that K is strongly normal, that E is a reflexive Banach space, that (X, d) is a complete K-metric space, that $P=\{d(x, y); x, y\in X\}$, that $\bar{P}$ denotes the weak closure of P, and that $P_1=\{z; z\in\bar{P}$ and $z\neq\mathcal{O}\}$.

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