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AN EXTREMAL PROBLEM ON THE CLASSICAL CARTAN DOMAINS

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1. This paper is concerned with the following extremal problem: Let D be a bounded domain in the $2n$-dimensional Euclidean space $\boldsymbol{C}^n$ of n complex variables $z=(z_1, \cdots, z_n)$. Denote by $\mathcal{F}(D)$ the family of holomorphic mappings from D into the unit hyperball B_n in $\boldsymbol{C}^n$. It is required to find the precise value

$$M(z_0, D)=\sup_{f\in\mathcal{F}(D)}\left|\det\left(\frac{\partial f}{\partial z}\right)_{z=z_0}\right| \qquad (z_0\in D),$$

where $\left(\frac{\partial f}{\partial z}\right)$ denotes the Jacobian matrix of f:

$$\left(\frac{\partial f}{\partial z}\right)=\begin{pmatrix} \frac{\partial f_1}{\partial z_1} \cdots \frac{\partial f_1}{\partial z_n} \\ \cdots\cdots\cdots\cdots \\ \frac{\partial f_n}{\partial z_1} \cdots \frac{\partial f_n}{\partial z_n} \end{pmatrix}, \qquad f=(f_1, \cdots, f_n).$$

If $w=h(z)$ is a biholomorphic mapping from D_1 onto D_2 and $w_0=h(z_0)$, then

$$M(z_0, D_1)=M(w_0, D_2)\left|\det\left(\frac{\partial h}{\partial z}\right)_{z=z_0}\right|,$$

namely, the quantity $M(z, D)$ is a relative invariant. Hence for a bounded homogeneous domain D it is sufficient to find the value $M(z_0, D)$ for a fixed point z_0 in D.

The automorphism of B_n which transforms a point $a=(a_1, \cdots, a_n)$ into the origin is given in the form

$$\varphi(z: a)=\mu(z-a)(I-\bar{a}'z)^{-1}U^{-1},$$

where $|\mu|^2=(1-a\bar{a}')^{-1}$ and $U'\bar{U}=(I-a'\bar{a})^{-1}$. Here I is the identity matrix and $\bar{A}$ denotes the conjugate matrix of A and A' the transposed matrix of A. Since

$$\left|\det\left(\frac{\partial\varphi}{\partial z}\right)_{z=a}\right|=(1-a\bar{a}')^{-(n+1)2}\geqq 1,$$

as far as $M(z_0, D)$ is concerned, we can replace $\mathcal{F}(D)$ by the subfamily $\mathcal{F}_{z_0}(D)$ of

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