A. YAMADA
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ON MARDEN'S UNIVERSAL CONSTANT OF FUCHSIAN GROUPS

BY AKIRA YAMADA

Let G be a Fuchsian group operating on the upper half plane H. For $z\in H$ and $r>0$, let $\Delta(z, r)$ be the open disc of radius r and center z. Define $G^{z,r}$ to be the subgroup of G generated by

$$I(z, r)=\{g\in G\,;\, d(z, gz)<2r\}=\{g\in G\,;\, \Delta(z, r)\cap\Delta(gz, r)\neq\phi\},$$

where $d(\cdot, \cdot)$ is the hyperbolic distance induced by the Poincaré metric $|dz|/\mathrm{Im}\, z$.

In this paper all references to distance, lines, discs, etc., will be with respect to the hyperbolic geometry unless otherwise stated.

Marden [6] proved the following:

THEOREM. *There is a constant $r>0$ such that, for any Fuchsian group G and $z\in H$, the subgroup $G^{z,r}$ is either cyclic or infinite dihedral (i.e. is generated by two elliptic transformations of order 2).*

Let $\mu(z, G)$ be the supremum of the set of constants r satisfying the conclusion of the Theorem. In fact, this is the maximum by discreteness. Set

$$\mu(G)=\inf_{z\in H}\mu(z, G) \quad \text{and} \quad \mu=\inf_{G}\mu(G).$$

μ will be called Marden's constant in this paper. The purpose of the paper is to determine Marden's constant explicitly. Our result is the following:

THEOREM 1. *For any Fuchsian group G we have*

$$\mu(G)\geqq\mu=\sinh^{-1}\sqrt{\frac{4\cos^2\pi/7-3}{8\cos\pi/7+7}}=0.131467\cdots$$

with equality occurring precisely when G is the (2, 3, 7) triangle group.

If we restrict ourselves to the case where G is torsion-free, then much better bound is obtained.

THEOREM 2. *For any torsion-free Fuchsian group G we have*

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