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RIEMANNIAN MANIFOLDS ADMITTING A PROJECTIVE VECTOR FIELD

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§ 1. Introduction.

Let M be a connected Riemannian manifold of dimension n covered by a system of coordinate neighborhoods $\{U; x^h\}$, where, here and in the sequel, the indices $h, i, j, k, \cdots$ run over the range $\{1, 2, \cdots, n\}$ and let g_{ji}, $\{_j{}^h{}_i\}$, ∇_j, $K_{kji}{}^h$, K_{ji} and K be respectively the metric tensor, the Christoffel symbols formed with g_{ji}, the operator of covariant differentiation with respect to $\{_j{}^h{}_i\}$, the curvature tensor, the Ricci tensor and the scalar curvature of M.

A vector field v^h on M is called a projective vector field if it satisfies

$$L_v\{_j{}^h{}_i\}=\nabla_j\nabla_i v^h+v^k K_{kji}{}^h=\delta_j^h\rho_i+\delta_i^h\rho_j \tag{1.1}$$

for a certain covariant vector field ρ_i, called the associated covariant vector field of v^h, where L_v denotes the operator of Lie derivation with respect to the vector field v^h. In particular, if ρ_i in (1.1) is zero vector field then the projective vector field v^h is called an affine vector field. When we refer in the sequel to a projective vector field v^h, we always mean by ρ_i the associated covariant vector field appearing in (1.1).

Recently, the present author [1, 2] proved a series of integral inequalities in a compact and orientable Riemannian manifold with constant scalar curvature admitting a projective vector field and then obtained necessary and sufficient conditions for such a Riemannian manifold to be isometric to a sphere.

The purpose of the present paper is to continue the work of the present author [1, 2] and to prove the following theorem.

Theorem A. *If a connected, compact, orientable and simply connected Riemannian manifold M with constant scalar curvature K of dimension $n>1$ admits a non-affine projective vector field v^h, then M is globally isometric to a sphere of radius $\sqrt{n(n-1)/K}$ in the Euclidean $(n+1)$-space.*

In the sequel, we need the following theorem due to Obata [3]. (See also [4].)

Theorem B. *Let M be a complete, connected and simply connected Rieman-*

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