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KODAI MATH. J.
3 (1980), 374—379

GROWTH OF A COMPOSITE FUNCTION OF ENTIRE FUNCTIONS

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§ 1. Introduction.

Let $f(z)$ and $g(z)$ be entire functions. Then we have the well-known inequality

$$\log M(r, f(g)) \leqq \log M(M(r, g), f). \tag{1}$$

And it follows from Clunie [2] that if $g(0)=0$, then for $r \geqq 0$

$$\log M(r, f(g)) \geqq \log M(c(\rho)M(\rho r, g), f), \tag{2}$$

where $0<\rho<1$ and $c(\rho)=(1-\rho)^2/4\rho$. Furthermore, these inequalities (1) and (2) are best possible. We next wish to have similar estimations of $T(r, f(g))$. As an immediate consequence of (1) and well-known inequalities $T(r, f) \leqq \log^+ M(r, f) \leqq 3T(2r, f)$, we have

$$T(r, f(g)) \leqq 3T(2M(r, g), f). \tag{3}$$

The inequality (3), however, is not sharp.

The main purpose of this paper is to give an upper estimation of $T(r, f(g))$ and prove the following:

THEOREM 1. *Let $f(z)$ and $g(z)$ be entire functions. If $M(r, g)>((2+\varepsilon)/\varepsilon)|g(0)|$ for any $\varepsilon>0$, then we have*

$$T(r, f(g)) \leqq (1+\varepsilon)T(M(r, g), f). \tag{4}$$

In particular, if $g(0)=0$, then

$$T(r, f(g)) \leqq T(M(r, g), f) \tag{5}$$

for all $r>0$.

Since $T(r, f(z^n))=T(r^n, f(z))$ for any meromorphic function $f(z)$, Theorem 1 is best possible. In the above example $g(z)$ is a polynomial. However, we shall

Received July 5, 1979