H. YAMADA
KODAI MATH. J.
3 (1980), 341—357

ON THE FIRST BOUNDARY VALUE PROBLEMS FOR SOME DEGENERATE SECOND ORDER ELLIPTIC DIFFERENTIAL EQUATIONS

BY HARUKI YAMADA

§ 1. Introduction.

Let $\Omega\subset \boldsymbol{R}^n$ be an bounded open domain with C^∞-boundary $\Sigma=\partial\Omega$. We consider the following so called "first boundary value problem".

$$[\mathrm{P}] \qquad \begin{cases} L(u)\equiv(a^{kj}u_{x_k})_{x_j}+b^k u_{x_k}+cu=f & \text{in } \Omega\,, \\ u|_{\Sigma}=0\,, \end{cases}$$

where $u_{x_k}=\partial u/\partial x_k$, etc., and the summation convention such as $(a^{kj}u_{x_k})_{x_j} = \sum_{k,j=1}^{n}(a^{kj}u_{x_k})_{x_j}$ are used. Without loss of generality, we can assume that $a^{kj}=a^{jk}$ (see the following condition [A.1]). Throughout this paper, we pose the following assumptions:

[A.1] $\qquad a^{kj},\ b^k,\ c,\ f\in C^\infty(\bar{\Omega})$ and real valued.

[A.2] $\qquad a^{kj}\xi_k\xi_j\geqq 0 \quad$ for all $(x,\ \xi)\in\bar{\Omega}\times\boldsymbol{R}^n$.

[A.3] $\qquad c<0\,, \quad c-b^k_{x_k}<0 \quad$ on $\bar{\Omega}$.

[A.4] $\qquad a^{kj}\xi_k\xi_j>0 \quad$ for all $(x,\ \xi)\in\Sigma\times(\boldsymbol{R}^n\backslash 0)$.

Under some additional assumptions for the boundary behavior of the coefficients replaced by [A.4], Kohn-Nirenberg [5], [6] and Oleinik [7], [8] proved several existence, uniqueness and regularity theorems for the problem [P]. For example, from their results we have the following theorem. (see e. g. [8] Chap. I, § 5).

THEOREM 1.1. *If in addition to* [A.1], [A.2], [A.3], *we assume that*

[A.5] $\qquad a^{kj}\nu_k\nu_j>0 \quad$ *for all* $x\in\Sigma$,

where $\nu=(\nu_1,\ \cdots,\ \nu_n)$ *is the unit normal vector for* Σ *at* x, *then for any* $f\in C^\infty(\bar{\Omega})$, *there is a uniquely determined weak solution* $u\in L^2(\Omega)$ *of* [P]. *Moreover, there*

Received June 5, 1979