H. NAKAGAWA
KODAI MATH. J.
3 (1980), 321-340

ON A CERTAIN MINIMAL IMMERSION OF A RIEMANNIAN MANIFOLD INTO A SPHERE

BY HISAO NAKAGAWA

Dedicated to Professor I. Mogi on his 60th birthday

Introduction. Minimal immersions of spheres into a sphere have been completely determined by do Carmo and Wallach [2]. Let $H^{r,n}$ be the space of all spherical harmonic polynomials of degree r on an n-dimensional sphere S^n, where $\dim H^{r,n}=(n+2r-1)(n+r-2)!/r!(n-1)! =: N(r)+1$. For an orthonormal basis $\{f_1, \cdots, f_{N(r)+1}\}$ of $H^{r,n}$, we define an immersion ι_r of S^n into an $(N(r)+1)$-dimensional Euclidean space $\boldsymbol{R}^{N(r)+1}$ by $\iota_r(x)=(f_1(x), \cdots, f_{N(r)+1})/(N(r)+1)^{1/2}$, which is called a *standard immersion*. Then the image by ι_r is contained in the unit sphere $S^{N(r)}(1)$ in $\boldsymbol{R}^{N(r)+1}$, and by means of a theorem of Takahashi [2] it is seen that ι_r is a minimal isometric immersion and $\iota_r(S^n)$ is not contained in the great sphere of $S^{N(r)}(1)$. With regard to the degree of the immersion in the sense of Wallach [9], they showed that the degree of ι_r is equal to r and if $r\leqq 3$, then ι_r is rigid.

On the other hand, Hong [3] introduced recently a notion of planar geodesic immersions. Let M and $\tilde{M}$ be complete connected Riemannian manifolds of dimension n and $n+p$, respectively. An isometric immersion ι of M into $\tilde{M}$ is called a *planar geodesic immersion* if each geodesic in M is mapped locally under the immersion into a 2-dimensional totally geodesic submanifold of $\tilde{M}$. Planar geodesic immersions of M into $S^{n+p}(c)$ have been classified by Little [5] and Sakamoto [8], independently, who stated that M is a compact symmetric space of rank one and the second fundamental form is parallel. The so-called Veronese manifold can be considered as one of examples determined by the planar geodesic immersion, while it can be regarded as the case of degree 2 in the ambient space.

When one pays attension to the rigidness of the standard immersion ι_r, it seems to be important to study the structure of the immersion with lower degree. As a matter of fact, the local version and the characterization of the Veronese manifold which is essentially an easiest model in our situation are investigated from variously different viewpoints. Furthermore, the local version concerning the immersion ι_3 of S^n into $S^{N(3)}$ has been treated by the author and Itoh [6]. In this paper, we shall be concerned with the characterization of the standard immersion ι_3 of S^n into $S^{N(3)}$. So as to do so, the notion of planar geodesic

Received May 12, 1979