BUI AN TON
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ON THE INITIAL-VALUE PROBLEM FOR COMPRESSIBLE FLUID FLOWS WITH VANISHING VISCOSITY

BY BUI AN TON

Introduction.

It is the aim of this paper to establish the existence of a unique local solution of the Cauchy problem for the fundamental system of equations describing the motion of viscous compressible heat conducting fluids. The solution obtained is analytic in the space-variables and converges in an appropriate sense to the solution of the limiting system as the viscosity tends to zero. We shall approximate the equations by a system of nonlinear equations of Sobolev-Galpern type. Estimates, in the quasi-norms of the generalized Gevrey spaces of Leray and Ohya are obtained by comparing solutions of some simple ordinary differential equations.

Let u be a vector-function and let ρ, θ be scalar functions. Consider the initial-value problem

$$(0.1)\qquad \begin{cases} \rho_\varepsilon\Big(\dfrac{\partial u_\varepsilon}{\partial t}+u_\varepsilon\cdot\nabla u_\varepsilon\Big)+\operatorname{grad}(\rho_\varepsilon+\theta_\varepsilon)-\varepsilon A u_\varepsilon=0\,, \\ \rho_\varepsilon\theta_\varepsilon\Big(\dfrac{\partial\theta_\varepsilon}{\partial t}+u_\varepsilon\cdot\operatorname{grad}\theta_\varepsilon-\rho_\varepsilon\cdot\operatorname{div} u_\varepsilon\Big)-\chi\Delta\theta_\varepsilon-\varepsilon B u_\varepsilon=0\,, \\ \dfrac{\partial\rho_\varepsilon}{\partial t}+\operatorname{div}(u_\varepsilon\rho_\varepsilon)=0\,,\quad \rho_\varepsilon(x,t) \text{ and } \theta_\varepsilon(x,t)>0 \text{ on } (0,T)\times R^3\,, \\ u_\varepsilon(x,0)=u_0(x)\,,\quad \rho_\varepsilon(x,0)=\rho_0(x) \text{ and } \theta_\varepsilon(x,0)=\theta_0(x) \text{ on } R^3\,. \end{cases}$$

A is the linear elliptic operator $Au=\Delta u+\operatorname{grad}(\operatorname{div} u)$ and B is the nonlinear operator $Bu=(\partial u_i/\partial x_k+\partial u_k/\partial x_i)^2$ with the usual summation convention.

The equations describe the motion of compressible fluids with viscosity ε. The velocity, the density and the absolute temperature of the fluid are denoted by u_ε, ρ_ε and θ_ε respectively. The coefficient of heat conduction is χ.

Unlike the case of incompressible fluids with constant density, there are few mathematical works on viscous compressible ones. In 1959, Serrin [9] proved a uniqueness theorem for solutions of (0.1) on bounded domains using the energy method. The problem of the existence of a local solution of (0.1)

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