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SEPARABLY HILBERTIAN FIELDS

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Let t and X be indeterminates. Let $f_i(t, X)$, $i=1, \cdots, m$ be irreducible polynomials over a field k and let $a(t)$ be a non-zero polynomial over k. A field k is called Hilbertian [6] if for any choice of f_i and a there exists an element s of k such that every $f_i(s, X)$ is irreducible and $a(s)\neq 0$. Any Hilbertian field of non-zero characteristic p is non-perfect because it has an element s such that X^p-s is irreducible. But this is not essential in applications of Hilbertian fields, and a slight modification of the definition allows us perfect Hilbertian fields. Let t and X be indeterminates. Let $f(t, X)$ be a polynomial over a field k such that it is separably irreducible over $k(t)$ as a polynomial of X. A field k is called separably Hilbertian if for any choice of such $f(t, X)$ it contains an element s such that $f(s, X)$ is separably irreducible over k. Let k be a Hilbertian field and let $f(t, X)$ be a polynomial over k which is separably irreducible with respect to X. Then the discriminant $D_f(t)$ is not zero. Now there exists an element s of k such that $f(s, X)$ is irreducible and $D_f(s)\neq 0$. Then $f(s, X)$ is separably irreducible, i.e., any Hilbertian field is separably Hilbertian. It has been known and will be shown below that two definitions are equivalent when the characteristic of a field k is zero. In the first section, it will be shown that a field k of non-zero characteristic is Hilbertian if and only if it is separably Hilbertian and non-perfect. In section 2, we will show some extensions of separably Hilbertian fields are also separably Hilbertian. Galois groups of extensions of separably Hilbertian fields of cohomological dimension 1 will be dealt in the last section. We will remark here an important application of Hilbertian fields essentially due to Lang [7] which does not seem to be well known. Let k be a field of characteristic p containing a finite field F_q. Let G be a connected linear algebraic group defined over F_q. Let x be a generic point of G over k. Then $k(x)$ is a finite Galois extension of $k(x^{-1}\cdot x^{(q)})$ with Galois group $G(F_q)$, the rational points of G over F_q. As $x^{-1}\cdot x^{(q)}$ is also a generic point of G over k, $k(x^{-1}\cdot x^{(q)})$ is isomorphic to $k(x)$ over k. This shows that if k is (separably) Hilbertian and if $k(x)$ is purely transcendental over k, k has a Galois extension with Galois group $G(F_q)$. It is known that $k(x)$ is purely transcendental if G splits over k. For example, let $\bar{F}_p$ be the algebraic closure of F_p and let t be an indeterminate. Then $\bar{F}_p(t)$ has a Galois extension with Galois group $G(F_q)$ for any connected linear

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