T. KASHIWADA
KODAI MATH. J
3 (1980), 70—82

ON V-HARMONIC FORMS IN COMPACT LOCALLY CONFORMAL KÄHLER MANIFOLDS WITH THE PARALLEL LEE FORM

BY TOYOKO KASHIWADA

Introduction. A locally conformal Kähler manifold (l. c. K-manifold) has been studied by I. Vaisman [8]. Especially when its Lee form is parallel, the manifold seems to have properties exceedingly similar to that of a Sasakian manifold. In this paper, we consider certain forms which correspond to C(C*)-harmonic forms of a Sasakian manifold and with it we have some informations on the Betti number of the manifold by a decomposition of such forms. The main result is that in a $2m$-dimensional compact l. c. K-manifold with the parallel Lee form, the following relation holds good between the p-th $(p<m)$ Betti number b_p and the dimension a_p of the vector space of certain p-forms which are defined in §2:

$$b_p=a_p-a_{p-2},$$

$$a_p=b_p+b_{p-2}+\cdots+b_{p-2r}, \qquad r=\left[\frac{p}{2}\right].$$

§1. **Preliminaries.** A locally conformal Kähler manifold is characterized as a Hermitian manifold $M^{2m}(\varphi, g)$, $2m$=the dimension, such that

$$\nabla_k\varphi_{ji}=-\alpha_j\varphi_{ki}+\alpha^r\varphi_{ri}g_{kj}-\alpha_i\varphi_{jk}+\alpha^r\varphi_{jr}g_{ki} \qquad (\varphi_{ji}=\varphi_j{}^r g_{ri})$$

with a closed 1-form α which is called the Lee form, ([2], [8]). Moreover, we assume $\nabla\alpha=0$, $|\alpha|=1$ and M is compact throughout this paper.

In this manifold, the following formulas are valid:

$$\nabla_k\varphi_{ji}=-\beta_j g_{ki}+\beta_i g_{kj}-\alpha_j\varphi_{ki}+\alpha_i\varphi_{kj}, \qquad \beta_j\overset{def}{=}\alpha^r\varphi_{rj},$$

$$J_{ji}\overset{def}{=}\nabla_j\beta_i=-\beta_j\alpha_i+\alpha_j\beta_i-\varphi_{ji} \qquad (=-\nabla_i\beta_j),$$

$$\alpha^r J_{ri}=\beta^r J_{ri}=0, \qquad J_i{}^r J_r{}^l=\beta_i\beta^l+\alpha_i\alpha^l-\delta_i{}^l,$$

$$\nabla_k\nabla_j\beta_i=-\beta^r R_{rkji}$$

$$=\beta_j g_{ki}-\beta_i g_{kj}+(\alpha_j\beta_i-\beta_j\alpha_i)\alpha_k,$$

Received January 23, 1979