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A BIHARMONIC NORMAL OPERATOR

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*ABSTRACT**. When a biharmonic singularity $s(x)$ is given on a boundary neighborhood A of a Riemannian manifold R, there arises a rather natural question about the biharmonic extendability of this singularity to $p(x)$ which is biharmonic on all of R. For harmonic singularities $s(x)\in H(A)$, the question was answered by L. Sario (1952), who showed that although $s(x)$ may not be harmonically extendable, nevertheless, in terms of the regular singularity $L(f)$, $s+L(f)$ is so extendable. Here, $L: \mathrm{C}(\partial A)\rightarrow H(A)$ is a bounded linear operator resembling the Dirichlet operator and is called normal. Analogously, in terms of $H^2(A)$, the set of biharmonic singularities on A, a biharmonic normal operator $L: \mathrm{C}(\partial A)\times \mathrm{C}(\partial A)\rightarrow H^2(A)$ is to resemble a Dirichlet operator and as an operator, is to be bounded. The purpose of the present effort is then to establish that, given a biharmonic normal operator L, each biharmonic singularity $s(x)$ has a biharmonic extension modulo a regular biharmonic function $L(f, g)$.

Examples may be obtained by applying the extension process to particular choices of L and $s(x)$; in particular, when $s(x)$ has a fundamental biharmonic singularity at a, then a biharmonic Green's function with singularity at a is obtained.

In his basic paper [4], L. Sario introduced a normal operator whose purpose was the construction of harmonic functions with certain prescribed behavior near the ideal boundary of a Riemann surface W. A full account of the applications of this operator, as well as an account of its own intrinsic interest, are given in, among others, the monograph of Rodin and Sario [1]. The issue of primary concern is, how does one accomplish showing that given a harmonic singularity $s(z)$ defined on a boundary neighborhood $W'\subset W$, there is a harmonic $p(z)$ defined on a Riemann surface W for which $p(z)-s(z)$ is a regular singularity $L(f)$. Here, in terms of α, the compact border of the bordered W', L is a continuous linear mapping similar to a Dirichlet operator. Explicitly, L is a linear mapping from $\mathrm{C}(\alpha)$ to the set of regular singularities $H_B(W')$; that is, the set of bounded harmonic functions with 0 flux. In this sense, $p(z)$ is said

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