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DETERMINISTIC LAWS OF TIME INHOMOGENEOUS DIFFUSION PROCESSES

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It is well known that a 1-dim. diffusion process whose paths are increasing is deterministic if the process is time homogeneous. The problem we will consider is to examine whether the same assertion is hold when the process is time inhomogeneous (cf. Orey [3]) and some sufficient conditions will be given.

1. Some Basic Properties.

Let (X_t) be a 1-dim. time inhomogeneous diffusion process with a transition probability $P(t, x; u, E)$, where the term "a diffusion process" means a continuous, strong Markov and conservative process. Then the process naturally defines the so called space-time process, which is a time homogeneous diffusion process on $[0, \infty)\times R^1$. If we denote it as (T_t, X_t), then its transition probability is given by

$$P_{(t,x)}[(T_s, X_s)\in A\times E]=P(t, x; t+s, E)I_A(t+s).$$

Let σ_y be the hitting time of (T_t, X_t) to $[0, \infty)\times[y, \infty)$, i.e.

$$\sigma_y=\inf\{s; X_s\in[y, \infty)\}$$

and define a function $u(t, x)$ by

$$u(t, x)=E_{(t,x)}[\exp(-\sigma_y)]. \tag{1}$$

Then we have

LEMMA 1. *The (Y_s) defined by*

$$Y_s=\exp(-s\wedge\sigma_y)u(t+s, X_{s\wedge\sigma_y}) \tag{2}$$

is a continuous $P_{(t,x)}$-martingale.

Proof. The martingale property is obtained by the direct caluculation. The continuity is a consequence of Th. 3.1 of Watanabe [2].

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