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HILBERT $B(H)$-MODULES AND STATIONARY PROCESSES

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1. Introduction.

In the theory of multivariate or Hilbert space valued stationary processes, the Gram matricial structure of the time domain of processes plays an important role. The Gram matricial structure of q-variate processes forms a module over a ring of all $q \times q$-matrices with a $q \times q$-matrix valued inner product, which can be seen as a Hilbert space over a matrix ring but not over the complex number field (cf. Masani [5]). Thus it is desirable to formulate such a structure abstractly free from the underlying probabilistic structure, as Kolmogorov first emphasized in 1940 for the univariate case where unspecified Hilbert spaces are prefered to L^2-spaces (cf. [4]). The purpose of the present paper is to establish such an abstact concept of the time domain of processes.

In the next section, we shall give definitions of Hilbert $B(H)$-modules and stationary processes on them. Our definition of a Hilbert $B(H)$-module is similar to the Paschke's definition [6] of inner product modules over B^*-algebras except that we require the range of the inner product, which we call the Gramian, is contained in the trace class. Such a requirement is always satisfied for the setting of q-variate or Hilbert space valued processes, and plays an essential role in our treatment. In Sect. 3 we shall study, in a general setting, positive sesquilinear maps valued in the predual of a W^*-algebra, example of which are the Gramian and the operator valued covariance function of stationary processes, and we shall examine the relation with the *-representation and construct a unitary representation which is a module version of Umegaki's construction [10], which are applied to the later sections. In Sect. 4 we shall show that the structure of Hilbert $B(H)$-modules is completely determined by the power of their modular bases, and that a Fourier expansion by the modular basis and Gramian is possible in a parallel way with one on the usual Hilbert spaces. In Sect. 5 applying a general theorem obtained in Sect. 3, the equivalence of stationary processes on Hilbert $B(H)$-modules is established by their covariance functions.

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