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KODAI MATH. J.
2 (1979), 346—361

# BOUNDARY VALUE PROBLEMS FOR REAL LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF FIRST ORDER

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## §1. Introduction.

Let $\Omega\subset R^n$ be a bounded connected open set with $C^\infty$-boundary $\Sigma=\partial\Omega$. We consider the following equation.

$$Lu=\sum_{i=1}^{n} b^i(x)\frac{\partial u}{\partial x_i}+c(x)u=f(x)\,, \tag{1.1}$$

where $b^i(x)$, $c(x)$, $f(x)\in C^\infty(\bar{\Omega})$ and all the functions are assumed to be real valued. We shall concern the problem: When, for suitably chosen closed subset $\Gamma\subseteqq\Sigma$, is the following statement true?

$[W_m]$ For any $f\in C^\infty(\bar{\Omega})$ and $g\in C^\infty(\Gamma)$, there is one and only one solution $u\in C^m(\bar{\Omega})$ of

$$Lu=f\,,\qquad u|_\Gamma=g\,,$$

where $m=1, 2, \cdots \infty$. Though, as is well known, the problem of solving first order partial differential equation (1.1) is reduced to solving system of first order ordinary difierential equations

$$\frac{dx_i}{dt}=b^i(x)\,,\qquad i=1, 2, \cdots n\,, \tag{1.2}$$

it is only a local result and our problems are related to global behavior of solution curves of (1.2). Our problems are also related to the first boundary value problems for second order equations with non-negative characteristic form investigated by Kohn-Nierenberg [2], Oleinik [3] and others. Though our equation (1.1) is a very special case of these degenerate second order equations, some of the difficulties in these problems are related to our problems and some meanings of conditions imposed to these problems are clarified by our settings.

We can pose the problem "under what conditions for $L$, $\Omega$ and $\Gamma$, is the statement $[W_m]$ true?" for general second order equations with not necessary non-negative characteristic forms. We shall not concern this problem in this note but show some simple examples at the end of this note.

Received June 2, 1978