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ON CRITERIA OF $\tilde{g}$-HYPERELLIPTICITY

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I. Introduction. Let S be a compact Riemann surface of genus $g \geqq 2$. S is called $\tilde{g}$-hyperelliptic provided that S is a two-sheeted covering of a surface of genus $\tilde{g}$. 0-hyperelliptic and 1-hyperelliptic are called hyperelliptic and elliptic-hyperelliptic, respectively. Let P be an arbitrary point of S. Let $\phi_1, \cdots, \phi_g$ be a basis of the space of abelian differentials of the first kind on S. Let k_i be the order of the zero of ϕ_i at P. Then we can choose $\phi_1, \cdots, \phi_g$ such that $0=k_1<k_2<\cdots<k_g \leqq 2g-2$. The sequence $G(P)=\{k_1+1, k_2+1, \cdots, k_g+1\}$ is called the Weierstrass gap sequence at P. P is called a Weierstrass point of S if $k_g \geqq g$. Denote $N(P)$ the sequence $\{1, 2, \cdots, 2g\}-G(P)$. If k is in $N(P)$, then there is a meromorphic function on S which is holomorphic except for a pole of order k at P.

It is well known that if $N(P)=\{2, 4, \cdots, 2g\}$ for some P, then S is hyperelliptic and vice versa. If S is elliptic-hyperelliptic and P is a fixed point of an elliptic-hyperelliptic involution, then $N(P)$ contains $\{4, 6, 8, \cdots 2g\}$ and no odd number less than $2g-3$ can be contained in $N(P)$. Moreover, if S is $\tilde{g}$-hyperelliptic, $g \geqq 4\tilde{g}-1$ and P is a fixed point of the $\tilde{g}$-hyperelliptic involution, then $l(P^{g-1})$ is equal to $(g+1)/2-\tilde{g}$ or $g/2-\tilde{g}$ [6]. Here, $l(P^{g-1})$ is the dimension of the space of meromorphic functions on S whose divisors are multiples of P^{1-g}. This is related directly with the vanishing property of the theta function at $K(P)$, the Riemann constant vector in the Jacobian variety.

In this paper we shall study some criteria of $\tilde{g}$-hyperellipticity in terms of a property of the Weierstrass gap sequence, which is also reflected with a vanishing property of the theta function. Accola [1] has treated a related problem in terms of the vanishing property at half periods of the Jacobian variety.

2. Statement of Theorems. We shall prove the following theorems.

THEOREM 1. *Let S be a compact Riemann surface of odd genus $g \geqq 11$. If $l(P^{g-1})=(g-1)/2$ for some point P on S, then S is elliptic-hyperelliptic.*

THEOREM 2. *Let S be a compact Riemann surface of even genus $g \geqq 14$. If $l(P^{g-1})=g/2-1$ for some point P on S, then S is elliptic-hyperelliptic.*

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