Y. MATSUYAMA
KODAI MATH. J.
2 (1979), 272—274

ON A CERTAIN HYPERSURFACES OF R^{2m+1}

BY YOSHIO MATSUYAMA

Introduction

It is a well-known theorem of Reeb (See [3], p. 25) that if a compact differentiable n-manifold M admits a Morse function with exactly two critical points, then M is a topological sphere.

Recently, Nomizu and Rodriguez [4] showed the following results as their geometric nature: Let M be a connected Riemannian $n(n\geqq 2)$-manifold isometrically immersed in a Euclidean m-space R^m and f its isometric immersion. Put $L_p(x)=(d(f(x), p))^2$ for $p\in R^m$, $x\in M$, where d is the Euclidean distance function. (a) If M is complete, and there exists a dense subset D of R^m such that every function of the form L_p, $p\in D$, has index 0 or n at any of its nondegenerate critical points, then M is totally umbilical in R^m, i.e., M is isometric to a Euclidean n-subspace or a Euclidean n-sphere in R^m. (b) If M is compact, and there exists a dense subset D of R^m such that every function of the form L_p, $p\in D$, has exactly two critical points, then M is isometric to a Euclidean n-sphere.

In the present paper we shall prove the following result.

THEOREM. *Let M be a connected, complete Riemannian $2m$ $(m\geqq 2)$-manifold isometrically immersed in R^{2m+1} with constant mean curvature. If there exists a dense subset D of R^{2m+1} such that every function of the form L_p, $p\in D$, has index 0, m or $2m$ at any of its nondegenerate critical points, then M is isometric to a Euclidean $2m$-subspace R^{2m}, a Euclidean $2m$-sphere S^{2m} in R^{2m+1} or the product $R^m\times S^m$ of an m-subspace R^m of R^{2m+1} and a sphere S^m in the Euclidean subspace perpendicular to R^m.*

When we consider the problem similar to (b) to obtain a result that M is isometric to S^{2m} or $R^m\times S^m$, it seems to be the natural condition that M is complete and there exists a dense subset D of R^{2m+1} such that every function of the form L_p, $p\in D$, has two critical points ([1], pp. 714-715).

1. Preliminaries

Let f be an isometric immersion of a connected Riemannian n-manifold M

Received February 7, 1978