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EXOTIC CHARACTERISTIC CLASSES OF CERTAIN Γ-FOLIATIONS

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§ 0. Introduction

In this paper we study characteristic classes and exotic characteristic classes [B] of foliations. We deal with a certain sort of Γ-foliations ($(G, \mathfrak{l})$-foliations) which is a generalization of Riemannian, projective and conformal foliations [NS, P]. The main purpose of this paper is to prove vanishing theorems for certain exotic characteristic classes of such Γ-foliations. As a step toward this, we obtain results which are relevant to strong vanishing theorems for characteristic classes of the Γ-foliations. These generalize the result of Nishikawa and Sato [NS]. In order to obtain these results, we use neither normal Cartan connections nor classifying spaces, but a product formula for secondary invariants [CS] and a technique used by Kobayashi and Ochiai in [KO].

Throughout this paper, all manifolds and mappings are assumed to be smooth (C^∞). In § 1, Chern-Weil theory of characteristic classes are reviewed, and the technique used in [KO] is slightly improved so that it may be applied to the case of foliations. In § 2, certain automorphisms of G-structures are specialized to l-automorphisms, which are generalized notions of affine, projective and conformal transformations, and then $(G, \mathfrak{l})$-foliations are defined. In § 3, so-called strong vanishing theorems for characteristic classes of $(G, \mathfrak{l})$-foliations are proved, where the results prepared in § 1 are applied. In § 4, we review some notions about exotic characteristic classes [B, H] such as cochain complexes WO_q and W_q, generalized characteristic homomorphisms for foliations and Vey-basis. In § 5, the vanishing theorems for certain exotic characteristic classes of $(G, \mathfrak{l})$-foliations are proved. In § 6, more detailed results are obtained in the case of projective and conformal foliations. Especially we find that all of the rigid exotic characteristic classes of conformal foliations vanish and the rest of the exotic characteristic classes coincide with the Godbillon-Vey invariant up to scalar multiples. In § 7, we prove the product formula and a derivative formula [H] for secondary invariants in a simple and unified manner.

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