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ON A CHARACTERIZATION OF THE EXPONENTIAL FUNCTION AND THE COSINE FUNCTION BY FACTORIZATION, III

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1. Introduction. This paper is a continuation of our previous one [1] with the same title, in which we proved the following fact.

THEOREM A. *Let $F(z)$ be an entire function, satisfying*

(a) $$F(z)=P_m(f_m(z))$$

with a polynomial P_m of degree m and an entire function f_m for $m=2^j$ (j: natural numbers) and $m=3$. Then

$$F(z)=A\cos\sqrt{H(z)}+B,$$

unless $F(z)=Ae^{H(z)}+B$. Here A, B are constants and H is an entire function.

In this paper we shall firstly consider the case that (a) holds for $m=2$, 4 and 3^j, where j runs over all natural numbers. Our theorem is the following.

THEOREM 1. *Let $F(z)$ be an entire function satisfying* (a) *for $m=2$, 4 and 3^j ($j=1, 2, \cdots$). Then*

$$F(z)=A\cos\sqrt{H(z)}+B,$$

unless $F(z)=Ae^{H}+B$. Here A, B and H are the same as in Theorem A.

The method of this paper gives more. Indeed (a) for i) $m=2, 3, 4$ and 5^j, or ii) $m=2, 3, 4, 7^j$ or iii) $m=2, 3, 4$, and 11^j implies the result, respectively.

2. Proof of Theorem 1. The first step, in which the case that

$$F(z)-b=A_2(f_2(z)-w_0)^2$$

has only finitely many zeros was considered in [1], gives the same conclusion, that is, $F(z)=Ae^{H(z)}+B$. Hence from now on we may assume that $F-b$ has infinitely many zeros and hence only infinitely many zeros of even order. The second step. Assuming that

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