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ON THE ZERO-ONE SET OF AN ENTIRE FUNCTION, II

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1. Introduction. Let $\{a_n\}$ and $\{b_n\}$ be two disjoint sequences with no finite limit points. If it is possible to construct an entire function f whose zeros are exactly $\{a_n\}$ and whose d-points are exactly $\{b_n\}$, the given pair $(\{a_n\}, \{b_n\})$ is called the zero-d set of f. Here of course $d \neq 0$. If further there exists only one entire function f, whose zero-d set is just the given pair $(\{a_n\}, \{b_n\})$, then the pair is called unique. It is well-known that unicity in this sense does not hold in general.

In this paper we shall prove the following

THEOREM. *Let $(\{a_n\}, \{b_n\})$ and $(\{a_n\}, \{c_n\})$ be the zero-one set and the zero-d set of an entire function N, where $d \neq 0, 1$. Then at least one of two given pairs is unique, unless N is an arbitrary entire function of the following form: $e^L + A$, where A is an arbitrary constant and L is an entire function.*

As a corollary we have the following fact.

COROLLARY. *Let N be an entire function with no finite lacunary value. Then every zero-d set of N excepting at most one is unique.*

Our proof depends on the impossibility of Borel's identity [1]. One of its form is the following

LEMMA. *Let $\{\alpha_j\}$ be a set of non-zero constant and $\{g_j\}$ a set of entire functions satisfying*

$$\sum_{j=1}^{p} \alpha_j g_j = 1 .$$

Then

$$\sum_{j=1}^{p} \delta(0, g_j) \leqq p-1 ,$$

where $\delta(0, g_j)$ denotes the Nevanlinna deficiency.

This form was stated in [2]. In our present case g_j is e^{L_j} and hence $\delta(0, g_j)=1$. Hence Lemma gives evidently a contradiction.

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