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KODAI MATH. J.
2 (1979), 171—186

ANTI-INVARIANT SUBMANIFOLDS OF A SASAKIAN SPACE FORM

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§1. Introduction.

In a previous paper [8] we studied anti-invariant submanifolds in a Kähler manifold, especially in a complex space form. In the present paper we shall study anti-invariant submanifolds of a Sasakian manifold, especially those of a Sasakian space form, in the same way as taken in [8].

An $(n+1)$-dimensional Riemannian manifold M isometrically immersed in a $(2m+1)$-dimensional Sasakian manifold $\bar{M}$ with structure tensors $(\phi, \xi, \eta, \bar{g})$ is said to be anti-invariant (resp. invariant) if $\phi T_p(M) \subset T_p(M)^{\perp}$ (resp. $\phi T_p(M) \subset T_p(M)$) for each point p of M, where $T_p(M)$ and $T_p(M)^{\perp}$ denote respectively the tangent and the normal spaces to M at p. Thus in an anti-invariant submanifold ϕX is normal to M for any vector X tangent to M. Since ϕ is necessarily of rank $2m$, we have $n \leqq (2m+1)-(n+1)$ which implies $n \leqq m$. In the present paper, we assume that for any anti-invriant submanifold M we consider the structure vector field ξ of the ambient manifold is tangent to M everywhere.

When for an anti-invariant submanifold M the structure vector field ξ of the ambient manifold $\bar{M}$ is tangent to M, then each of the following assumptions (a), (b), (c) is not meaningful: (a) the second fundamental form is parallel; (b) the mean curvature vector is parallel; (c) the connection induced in the normal bundle is flat. So, in the present paper, we shall replace the assumptions (a), (b), (c) respectively by new but rather weaker assumptions (a′), (b′), (c′) as follows: (a′) the second fundamental form is pseudo-parallel; (b′) the mean curvature vector is pseudo-parallel; (c′) the connection induced in the normal bundle is pseudo-flat (see Lemmas 3.2, 3.3 and 4.1).

§2. Sasakian manifolds.

First, we would like to recall definitions and some fundamental properties of Sasakian manifolds. Let $\bar{M}$ be a $(2m+1)$-dimensional differentiable manifold of class C^{∞} and ϕ, ξ, η be a tensor field of type $(1, 1)$, a vector field, a 1-form on $\bar{M}$ respectively such that

Received December 17, 1977