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HIGHLY CONNECTED POINCARÉ COMPLEXES

Dedicated to Professor A. Komatu on his 70th birthday

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Introduction.

We are interested in the following problem proposed by Wall in [2] "Classify up to homotopy $(n-1)$-connected Poincaré complexes of dimension $2n+1$ and $2n+2$."

In this paper we shall discuss the case of dimension $2n+2$ under some additional conditions. Let K be a Poincaré complex which is $(n-1)$-connected and of dimension $2n+2$. If K has the same rational homology as the sphere, then the homology $H_*(K;Z)$ is as follows

$$H_0(K;Z)=Z=H_{2n+2}(K;Z)$$

$$H_n(K;Z)=G=H_{n+1}(K;Z)$$

$$H_i(K;Z)=0 \qquad \text{for other dimensions,}$$

where G denotes a finite abelian group. We denote by $P(n, n+1;G)$ the complex K such as above and call it a Poincaré complex of type $(n, n+1;G)$. Then our main results are

THEOREM A. *Let $n\geqq 3$ and $G\otimes Z_2=0$. Then $P(n, n+1;G)$ has the same homotopy type as the connected sum of $P(n, n+1;G_1)$ and $P(n, n+1;G_2)$ if G is a direct sum of G_1 and G_2.*

THEOREM B. *Under the same conditions as Theorem* A, *if $P(n, n+1;G)$ is S-reducible it's homotopy type is unique with respect to n and G.*

By applying these theorems to the case of manifolds we shall prove

THEOREM C. *Let M be a $(n-1)$-connected rational homology sphere which is a smooth manifold of dimension $2n+2$ with no 2-torsion. Then M is uniquely determined up to homotopy by homology for $n\equiv 0, 1$ mod 4.*

The case of $G\otimes Z_2\neq 0$ (essencially, G is a 2-group) is more complicated, therefore we shall discuss it in the subsequent paper.

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