

ON SOME DIFFERENTIAL GEOMETRIC CHARACTERIZATIONS OF A BUNDLE-LIKE METRIC

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§1. Introduction.

We have known some interesting theorems about the behaviour of geodesics of a bundle-like metric :

THEOREM A (Y. Muto [6]). *A geodesic of a fibred riemannian manifold tangent to an “allowed curve” at one point is always an “allowed curve” if and only if “the fibres are parallel”.*

THEOREM B (B.L. Reinhart [10]). *A geodesic of a bundle-like metric is orthogonal at one point if and only if it is orthogonal at every point.*

The topological obstructions for the existence of the foliation with bundle-like metric were studied by R. Sacksteder [11], J. S. Pasternack [8, 9] and others. But of the conditions for the given riemannian metric of a foliated manifold to be a bundle-like metric very little is definitely known (cf. [5]). Of course, not all foliations have bundle-like metrics [10]. The completeness of a bundle-like metric was studied by one of the authors [3, 4].

In this note, we will give some differential geometric conditions for a given riemannian metric \langle , \rangle on a foliated riemannian manifold of codimension one to be a bundle-like metric in terms of geodesics. Our main theorem is the following :

THEOREM C. *Suppose that a foliated riemannian manifold is of codimension one and that all leaves are totally geodesic with respect to the given riemannian metric \langle , \rangle . Then the metric \langle , \rangle is a bundle-like metric with respect to the foliation if and only if all geodesics with “angle α ” to a leaf at one point have “constant angle α ” to each leaf at every point.*

Furthermore, we can give a proof of the following theorem :

THEOREM D. *The given riemannian metric \langle , \rangle on a foliated riemannian manifold of codimension one is a bundle-like metric with respect to the foliation if and only if all geodesics orthogonal to a leaf at one point are orthogonal to each leaf at every point.*

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