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ON PARALLEL CONFORMAL CONNECTIONS

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Introduction. Riemannian manifolds endowed with a parallel conformal connection $\nabla_{p.c}$ have been defined by the present author in [1]. In this paper one studies in the first section a type of such manifolds for which the principal field X associated with $\nabla_{p.c}$ is *parallel*. In this case X is an *infinitesimal homothety* of the volume element of M_c and is an invariant section of the cannonical form in the set of 2-frames $\mathcal{O}^2(M_c)$. If M_c is of even dimension $2m$, then the connection $\nabla_{p.c}$ defines on M_c a *conformal symplectic form* φ and the dual field of the principal 1-form α (α is the dual form of X with respect to the metric of M_c) with respect to φ is a *Killing field*. Finally it is shown that M_c is of constant scalar curvature and is *Ricci flat* in the direction of X. In the second section, making use of some notions introduced by K. Yano and S. Ishihara in [5] and by J. Klein in [7] one studies different properties of the tangent bundle manifold TM_c. Thus the *complete lift* φ^c of φ, on TM_c is a homogenous form of degree 1 and is *also conformal symplectic*. If V is the *canonical field* on TM_c, then the Lie bracket $[V, X]$ is an *infinitesimal automorphism of* φ^c. Further some properties involving the *canonical symplectic form* Ω on TM_c (Ω is a *Finslerian form*) and a second conformal symplectic form Θ, which is homogenous of degree 2, are discussed. In the last section one considers a regular *mechanical system* (in the sense of J. Klein [8]), $\mathcal{M}=\{M_c, T, \pi\}$ such that the *kinetic energy* T is homogenous of degree 2 and the *dynamical system* Z associated with $\mathcal{M}$ is a *spray* on M_c.

1. M_c **manifold.** Let M be an n-dimensional C^∞-Riemannian manifold and let $\mathcal{O}(M)$ be the bundle of orthonormal frames of M. If $\mathcal{O}\in\mathcal{O}(M)$ is such a frame, let $\{e_i\}$, $\{\omega^i\}$ and $\omega^i_k=\mathcal{G}^i_{kj}\omega^j$, $i, k, j=1, \cdots, n$, be the vectorial and dual basis and the connection forms associated with $\mathcal{O}$ respectively. Then the line element dp ($p\in M$), the connection equations and the structure equations (E. Cartan) are respectively

$$dp=\omega^i\otimes e_i\,, \tag{1.1}$$

$$\nabla e_i=\omega^k_i\otimes e_k\,, \tag{1.2}$$

$$d\wedge\omega^i=\Omega^i+\omega^k\wedge\omega^i_k\,, \tag{1.3}$$

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