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REAL SUBMANIFOLDS IN A QUATERNIONIC PROJECTIVE SPACE

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§ 1. Introduction

The second fundamental form plays a very important role in the study of submanifolds, cf. [1]. From this point of view J. Simons established in [12] a formula for the Laplacian of the length of the second fundamental form, which has enabled us to have a lot of global informations on submanifolds. (See for example [2] and [7].)

On the other hand H. B. Lawson, Jr. introduced in [6] the notion of generalized equators $M^C_{q,s}$ and $M^Q_{q,s}$ of complex and quaternionic projective spaces, which have stimulated the study of real hypersurfaces of complex and quaternionic projective spaces. (See for example [8] and [11].) His idea, which heavily depends on the theory of fibrations formulated by B. O'Neill in [10], is first to construct a circle bundle over a real hypersurface of the projective space by using the fibrations which are compatible with the Hopf fibrations and second to project known theorems for hypersurfaces of the sphere down to real hypersurfaces of the projective space.

The study of submanifolds of codimension>1 of Kählerian and quaternionic Kählerian manifolds is recently started by using Simons' formula. Some of the examples of those submanifolds are invariant submanifolds, totally real (or anti-invariant) submanifolds and totally complex submanifolds. (See for example [3] and [13].) Okumura introduced in [9] the notion of anti-holomorphic submanifolds in studying real submanifolds of codimension>1 in a Kählerian manifold using the Hopf fibration.

The purpose of this paper is to characterize a real submanifold M in a quaternionic projective space QP^m by investigating a submanifold $\bar{M}$ in the unit sphere S^{4m+3}, where $\bar{M}=\pi^{-1}(M)$ and the fibration $\pi : \bar{M} \to M$ is compatible with the Hopf fibration $\tilde{\pi}: S^{4m+3} \to QP^m$. In § 2 we prepare fundamental formulas for submanifolds in a quaternionic projective space. In § 3 we review the theory of fibrations, then establish some relations between the connections in the normal bundles of M in QP^m and of $\bar{M}$ in S^{4m+3}. In § 4 we focus our attention to the properties derived from the second fundamental tensors of M and $\bar{M}$. Finally in § 5 we deal with a special class of real submanifolds, called antiquaternionic, in a quaternionic projective space QP^m, cf. Definition 5.1, and prove a certain

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