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WEAKLY MIXING PROPERTIES OF SEMIGROUPS OF LINEAR OPERATORS

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Introduction.

The concepts of strong and weak mixing play an important role in the theory of measure-preserving transformations. The strongly mixing condition was connected with the mean ergodic theorem by Blum and Hanson [3], Brunel and Keane [4], and Hanson and Pledger [7], and was generalized to transformations in infinite measure spaces by Krengel and Sucheston [17]. The strongly mixing properties of linear operators of L_1-spaces have been investigated by Lin [18], Akcoglu and Sucheston [1, 2], Sato [21], and Fong and Sucheston [6]. On the other hand, the weakly mixing properties have been generalized to linear operators on general Banach spaces in connection with the mean ergodic theorem by Jones [11, 12, 13], Nagel [20], and Jones and Lin [14].

Let T be a linear operator on a Banach space E. A typical condition meaning strong mixing of T is stated as follows: for each $x\in E$, $T^n x$ converges weakly. A corresponding condition of weak mixing is as follows: for each $x\in E$, there exists a subsequence $\{n_k\}$ of density 1 such that $T^{n_k}x$ converges weakly.

In this paper, we shall consider the weakly mixing properties of discrete cyclic semigroups and one parameter semigroups of linear operators on Banach spaces. § 1 contains some preliminaries concerning upper and lower densities. In § 2 we shall present results concerning the weakly mixing properties of semigroups on Banach spaces. We shall introduce several conditions meaning the weakly mixing properties and including the conditions given in [11], [13], [14], and [20]. Among those conditions, we shall obtain a number of implications. In § 3 we shall give further results for operator convergence of weak mixing type. In § 4 we shall consider semigroups of positive linear operators on AL-spaces and strengthen theorems in § 3.

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§ 1. Preliminaries.

Let J be a subset of the positive integers $Z^+=\{1, 2, \cdots\}$, and let $|J|$ denote

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